Reg. No ..... Name **22U322**

B. Sc DEGREE END SEMESTER EXAMINATION : OCTOBER 2022

SEMESTER 3 : COMPUTER APPLICATIONS

COURSE : **19U3CRCMT3 : CALCULUS**

(For Regular - 2021 Admission and Improvement / Supplementary - 2020 / 2019 Admission)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Prove that the radius of curvature at any point of the catenary $y = c\ \cos h\ \left(\frac{x}{c}\right)$ varies as the square of the ordinate.
2. Determine the constants a and b so that the curve $y = x^3 + ax^2 + bx$ has an inflecton at the point (3, -9).
3. $Find\ the\ n^{th}\ derivative\ of\ \sin\ x\ \cos\ 3x.$
4. Find $\frac{\partial f}{\partial x}\ and\ \frac{\partial f}{\partial y}\ if\ f\Big(x,y\Big)\ = \tan^{-1}\left(\frac{y}{x}\right).$
5. Use chain rule to find the derivative of $w = 2ye^x -$ ln z with respect to t along the path x = ln$(t^2+1), y =$ tan^{-1}t,$z = e^t$.
6. $If\ f\big(x,y\big) = x^2y - 2xy\ and\ \ R:\ 0 \le x \le 3,\ \ -2 \le y \le 0,\ then\ evaluate\ \iint_R f\big(x,y\big)dA.$
7. Find the centroid of the region R between the semi-circle $y = \sqrt{a^2 - x^2}$and the x-axis.
8. Find the area between the curves y= sec^2 x and y= sin x from 0 to $\pi/4$.
9. Evaluate the integral $\int_0^{\frac{\pi}{3}}\ \frac{\tan\theta\ d\theta}{\sqrt{2\ sec\ \theta}}$.
10. Evaluate $\int_0^3\ \int_0^2\ \left(4 - y^2\right)\ dy\ dx.$
11. Find the Jacobian J(u,v) for the transformation x = u cosv, y = u sinv .
12. Integrate f (x, y) = x/y over the region in the first quadrant bounded by the lines y = x, y = 2x, x = 1, x = 2.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Expand ln cosh x by Maclaurin's series.
14. Find the co-ordinates of the centre of curvature at a point (x,y) of the parabola $y^2 = 4ax.$
15. Find all local maxima,local minima and saddle points of the function $f\big(x,y\big) = 2xy - 5x^2 - 2y^2 + 4x + 4y - 4.$
16. $If\ v = \ln\Big(x^3 + y^3 + z^3 - 3xyz\Big),\ show\ that\ \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} = \frac{3}{x+y+z}.$
17. Find the area of the region enclosed by the curve $y^2 = 4x\ and\ the\ line\ y = 4x - 2.$
18. Find the length of the curve y = log sec x between points given by x = 0 and x = $\frac{\pi}{3}$
19. Change the cartesian integral into equivalent polar integral and hence evaluate $\int_0^1\ \int_0^{\sqrt{1-x^2}}\ \left(x^2 + y^2\right)\ dy\ dx.$
20. Evaluate $\int_0^a\ \int_0^x\ \int_0^y\ xyz\ dz\ dy\ dx.$

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Find all asymptotes of the curve $y^3 - 5xy^2 + 8x^2y - 4x^3 - 3y^2 + 9xy - 6x^2 + 2y - 2x - 1 = 0.$
22. Find the absolute maximum and minimum values of $f\big(x,y\big) = x^2 + xy + y^2 - 6x + 2$ on the rectangular plate $0 \le x \le 5,\ \ -3 \le y \le 0.$

23. a) Find the area of the surface generated by revolving the curve $y = \sqrt{2x+1}, \quad 0 \leq x \leq 3$, about the x-axis.
b) Find the volume of the solid generated by revolving the region bounded by the x-axis,the curve y = $3x^4$ and the lines x = 1 and x = -1 about the line y = 3.

24. Find the volume of the portion of the solid sphere $\rho \leq a$ that lies between the planes $\phi = \frac{\pi}{3} and\ \phi = \frac{2\pi}{3}$.

(10 x 3 = 30)