Reg. No .... Name **22U321**

B. Sc. DEGREE END SEMESTER EXAMINATION : OCTOBER 2022

SEMESTER 3 : MATHEMATICS

COURSE : **19U3CRMAT03 : VECTOR CALCULUS, THEORY OF EQUATIONS AND MATRICES**

(For Regular - 2021 Admission and Improvement / Supplementary - 2020 / 2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A
Answer any 10 (2 marks each)

1. Define eigen value and eigen vector.
2. State Gauss divergence theorem and prove that for any closed surface S, $\iint_s curl\ \bar{F}.\hat{n}ds = 0.$
3. Define homogeneous equation and give the condition for trivial solution and non trivial solution.
4. Calculate the eigen values of matrix $\begin{bmatrix} 1 & 6 \\ 3 & 7 \end{bmatrix}$
5. Find the sum of the cubes of the roots of the equation $x^3 - 2x^2 + x + 1 = 0$
6. Explain the term normal form of a matrix with examples.
7. Form a rational quartic equation whose roots are 1,-1 and $2 + \sqrt{3}$
8. Show that $div(grad\phi) = \nabla^2\phi$.
9. If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$, find the value of $\sum \alpha^2$.
10. Evaluate the angle between the normals to the surface $xy = z^2$ at the points (4,1,2) and (3,3,-3).
11. Show that $\nabla\left(\overline{A} \times \overline{B}\right) = \left(\nabla.\overline{B}\right)\overline{A} - \left(\nabla.\overline{A}\right)\overline{B} + \left(\overline{B}.\nabla\right)\overline{A} - \left(\overline{A}.\nabla\right)\overline{B}.$
12. State Stoke's theorem.

(2 x 10 = 20)

PART B
Answer any 5 (5 marks each)

13. If $\overline{F} = (2x^2 - 3z)i - 2xyj - 4xk$, then evaluate $\iiint_V \nabla \times \overline{F}\ dv,$ where V is the closed region bounded by the planes $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4.$
14. Find the values of constants λ and μ so that the surface $\lambda x^2 - \mu yz = (\lambda + 2)x$ and $4x^2y + z^3 = 4$ may intersect orthogonally at (1,-1,2)
15. Find the eigen values of $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ and evaluate the eigen vector corresponding to the larger eigen value .
16. Solve the equation $x^3 + x^2 - 16x + 20 = 0$, given that some of its roots are repeated.
17. Evaluate $\int_c \left(2x^2y + y + z^2\right)i + 2\left(1 + yz^2\right)j + \left(2z + 3y^2z^2\right)k.\,d\bar{r}$ along the curve $c : y^2 + z^2 = a^2, x = 0.$
18. Use Gauss Jordan method to find the inverse of $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$.

19. Given that the roots of the equation $4x^3 - 24x^2 + 23x + 18 = 0$ are in arithmetic progression, solve the equation.

20. Evaluate grad ϕ when $\phi = 3x^2y - y^3z^2$ at the point (1,-2,-1)

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Evaluate $\iint_s \overline{A}.\hat{n}ds$, where $\overline{A} = (x+y^2)i - 2xj + 2yzk$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant.

22. (a) Solve $x^4 + 12x^2 + 8x + 6 = 0$ using Ferrari's method
(b) Solve $x^3 - 6x^2 + 3x - 2 = 0$ using Cardan's method.

23. Verify Cayley Hamilton theorem for the matrix A and find A^{-1}, where A = $\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

24. (a) Prove that $curl(curl\bar{v})$ = $grad(div\bar{v})$ = $\nabla^2\bar{v}$.
(b) If $\bar{A}(3xz^2)i - (yz)j + (x+2z)k$, find $curl(curl\bar{A})$

(10 x 3 = 30)