Reg. No…….. Name…… **22U521**

B. Sc. DEGREE END SEMESTER EXAMINATION - OCTOBER 2022

SEMESTER **5 : MATHEMATICS (CORE COURSE)**

COURSE : **19U5CRMAT06: DIFFERENTIAL EQUATIONS**

(For Regular - 2020 Admission & Supplementary - 2019 Admission)

Time: Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Define exact differential equation.
2. Check whether the equation $(e^y + 1)Cos\ x\ dx + e^y Sin\ x\ dy = 0$ is exact.
3. Find the integrating factor of $x^2y\ dx - (x^3 + y^3)\ dy = 0$
4. Check whether e^x and e^{-x} are linearly independent.
5. Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$
6. What is the solution of a linear differential equation if the roots of auxiliary equation is 1+i, 1-i, 1+i, 1-i
7. Write the normalized form of the differential equation $(x-5)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + \frac{1}{x}y = 0$
8. Find the singular points of $(3x+1)x\ \frac{d^2y}{dx^2} - (x+1)\frac{dy}{dx} + 2y = 0$
9. Define regular singular point of the differential equation.
10. Write the auxiliary equation of $z(px - qy) = y^2 + x^2$
11. Eliminate the constants a and b from $x^2 + y^2 + (z-c)^2 = a^2$
12. Eliminate the arbitrary function from the equation and form the partial differential equation of $z = xy + f(x^2 + y^2)$.

$(2 \times 10 = 20)$

Part B

Answer any 5 (5 marks each)

13. Solve $(3xy^2 - y^3)dx - (2x^2y - xy^2)dy = 0$
14. Find the orthogonal trajectories of the family of parabolas $y^2 = 4ax$
15. Given that $y = x$ is a solution of $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = 0$, then find a linearly independent solution by reducing the order. Write the general solution.
16. Find the general solution of $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\ Sin\ x$

17. Find the power series solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$.
18. Locate and classify singular points on the x-axis for the differential equation

    $x^3(x-1)\frac{d^2y}{dx^2} - 2(x-1)\frac{dy}{dx} + 3xy = 0$
19. Form the partial differential equation by eliminating function f from the equation $z = f(\frac{xy}{z})$
20. Find the integral curves of $\frac{dx}{xz-y} = \frac{dy}{yz-x} = \frac{dz}{1-z^2}$

$(5 \times 5 = 25)$

Part C

Answer any 3 (10 marks each)

21. Solve $(x^2 + y^2 + 2x)\,dx + 2y\,dy = 0$
22. Find the general solution of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2 + e^x + 2xe^x + 4e^{3x}$
23. Find the Frobenius series solutions for the differential equation

    $2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x-5)y = 0$ in some interval $0 < x < \mathbb{R}$
24. Find the general integrals of the following partial differential equation

    (a) $x^2p + y^2q = (x+y)z$

    (b) $x(y^2+z)p - y(x^2+z)q = z(x^2-y^2)$

$(10 \times 3 = 30)$