22U538-S

Reg. No………………………………………….   Name……………………………………………

B.Sc. DEGREE END SEMESTER EXAMINATION: OCTOBER 2022

SEMESTER **5: MATHEMATICS (CORE COURSE)**

COURSE: **15U5CRMAT7: ABSTRACT ALGEBRA**

(Common for Supplementary 2015/2016/2017/2018 admissions)

Time: Three Hours   Max. Marks: 75

PART A
Each Question carries 1 Mark
Answer All Questions

1. How many elements are there in the ring of matrices $M_2(\mathbb{Z}_3)$?
2. Prove that the set G = {0,1,2,3,4,5} is an abelian group under addition modulo 6.
3. Prove that a cyclic group of order eight is homomorphic to a cyclic group of order four.
4. Give an example for a non-commutative ring.
5. Prove that every permutation in S_n can be written as a product of at most (n-1) transpositions for n ≥ 2.
6. Express the additive inverse of 21 in the group $< \mathbb{Z}_{73}, +_{73} >$ as a positive integer in $\mathbb{Z}_{73}$.
7. State true or false: Null set forms a group.
8. Define canonical map from $\mathbb{Z}$ to $\mathbb{Z}/n\mathbb{Z}$
9. Which are the zero divisors in $\mathbb{Z}_{12}$?
10. Define a division ring.   (1 x 10 =10)

PART B
Each Question carries 2 Marks
Answer any Eight

11. Show that arbitrary intersection of subgroups of a group *G* is a subgroup of *G*.
12. Prove that every cyclic group is abelian.
13. For given subrings $U1$ and $U2$ of a ring R, show that their intersection $U1 \cap U2$ is also a subring of R.
14. Is -1 the generator of the cyclic group $\mathbb{Z}$? If yes, describe how to generate 3 using the the generator -1.
15. Show that if $a \in G$, where G is a finite group with identity e, then there exist a positive integer n such that $a^n = e$.
16. Define Klein - 4 group and draw its group table.
17. Determine all ideals of $\mathbb{Z} \times \mathbb{Z}$.
18. If G is a finite group of even order then prove that there exists at least one element $a \neq e$ where e is the identity element, such that $a = a^{-1}$.
19. Prove: Every group of prime order is cyclic.
20. Let f=(1 4 3 2 5) and g = (1 2)(4 3 5)$\in S_5$ find fog and gof.

(2 x 8 =16)

PART C
Each Question carries 5 Marks
Answer Any Five

21. Draw the group table of a cyclic group of order 5.
22. Define an automorphism of a group. Show that all automorphisms of a group G form a group under function composition.
23. State and prove characterisation of maximal normal subgroups.
24. Let $G=\{1,a,a^2,a^3\}$ $(a^4=1)$ be a group and $H=\{1,a^2\}$ is a subgroup of G under multiplication. Find all cosets of H.
25. If ‘p’ is prime prove that $\mathbb{Z}p$ is a field.
26. Prove: If a ring R can be partitioned into cells with both the induced operations well defined and if the cells form a ring under these induced operations, then the cell containing additive identity 0 of R will be a subgroup N of the additive group $(R, +)$. Furthermore, $\forall\ r \in R$ and $\forall\ n \in N$, both $rn \in N$ and $nr \in N$.
27. Let S be the set of all real numbers except -1. Define * on S by $a * b = a + b + ab$. Show that (S, *) forms a group. (5 x 5 = 25)

PART D
Each Question carries 12 Marks
Answer Any Two

28. State and prove fundamental theorem for group homomorphism.
29. State and prove Cayley’s Theorem.
30. Prove: No permutation of a finite set can be expressed both as a product of an even number of transpositions and as a product of an odd number of transpositions.
31. State and prove any necessary and sufficient condition for any subset of a group to become a subgroup under the same group operation. (12 x 2 = 24)