Reg. No………………………………………….. Name…………………………………………. **22U544**

B.Sc. DEGREE END SEMESTER EXAMINATION : OCTOBER 2022

SEMESTER **5 : COMPUTER APPLICATION**

COURSE: **19U5CRCMT6 : MATHEMATICAL ANALYSIS**

(*For Regular - 2020 Admission and Supplementary - 2019 Admission)*

Time: Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks)

1. Define an interval. Give an example of a closed interval.
2. Find the infimum and the supremum of $\left\{1+\frac{(-1)^n}{n}: n \in \mathbb{N}\right\}$.
3. Every supremum of a set is a greatest member. True/False. Justify your answer.
4. Show that the set of natural numbers is order complete.
5. Define interior of a set. What is the interior of $\left\{1, \frac{1}{2}, \frac{1}{3}, \dots \dots \dots .\right\}$.
6. Define a derived set. Obtain the derived set of $\{x: 0 < x < 1; x \in \mathbb{Q}\}$.
7. Define a countable set. Give an example.
8. Show that the set $\left\{1, -1, 1\frac{1}{2}, -1\frac{1}{2}, 1\frac{1}{3}, -1\frac{1}{3}, \dots \dots \dots .\right\}$ is closed, but not open.
9. Find the limit points of the sequence $\{S_n\}$ where $S_n = 1 + (-1)^n; n \in \mathbb{N}$.
10. Define a bounded sequence. Give an example.
11. What is the nature of convergence of sequence $\{1, 2, \frac{1}{2}, 3, \frac{1}{3}, \dots \dots \dots\}$.
12. Show that $Re(iz) = -Im(z)$.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. If a be a positive real number and b, any real number, then prove that there exists a positive integer n such that $na > b$.
14. Prove that every open interval (a, b) contains a rational number.
15. Prove that the union of two closed sets is a closed set.
16. Prove that the derived set of a bounded set is bounded.
17. Find limit inferior and limit superior of the sequence $\{a_n\}$ where $a_n = \sin\frac{n\pi}{3}; n \in \mathbb{N}$.
18. Prove that every convergent sequence is bounded.
19. Show that the sequence $\{b_n\}$ where $b_n = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \frac{1}{(n+3)^2} + \dots \dots \dots \dots + \frac{1}{(2n)^2}$ converges to 0.
20. Show that
    a) $\left|e^{i\theta}\right| = 1$
    b) $\overline{e^{i\theta}} = e^{-i\theta}$

(5 x 5 = 25)

PART C

Answer any 3 (Each one carries 10 marks)

21. Prove that the set of rational numbers is not order complete.
22. Prove that the derived set S' of a bounded infinite set $S \subseteq \mathbb{R}$ has the smallest and the greatest members.
23. State and prove Cauchy's general principle of convergence.
24.
    a) Find the cube roots of the complex number $-8i$.
    b) If a set contains each of its accumulation points, then prove that this set must be a closed set.

(10 x 3 = 30)

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