Reg. No………………………………………….. Name…………………………………………… **22U538**

B. Sc. DEGREE END SEMESTER EXAMINATION : OCTOBER 2022

SEMESTER **5: MATHEMATICS**

COURSE: **19U5CRMAT07 : ALGEBRA**

(*For Regular - 2020 Admission and Supplementary - 2019 Admission)*

Time: Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Say True or False ' If $*$ is a binary operation on any set G, then $a * a = a$, for all $a \in G$.
2. Let S be the set of all real numbers except -1, define a binary operation $*$ on S as $a * b = a + b + ab$ then find the solution for the equation $2 * x * 3 = 7$.
3. Find the number of cyclic subgroups of $U(10)$.
4. If $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{pmatrix}$ find α^{-1}.
5. Prove that every group of prime order is cyclic.
6. Express the following permutation α as product of disjoint cycles and product of transpositions, where $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 4 & 1 & 8 & 2 & 5 & 7 \end{pmatrix}$. Also find O($\alpha$).
7. Let $\mathbb{R}^*$ be the group of nonzero real numbers under multiplication, $\varphi: \mathbb{R}^* \to \mathbb{R}^*$, defined by $\varphi(x) = |x|$, prove φ a homomorphism. Also find $Ker\varphi$.
8. Define a normal subgroup with example.
9. Say true or false $\mathbb{Z}_3 \times \mathbb{Z}_8$ is isomorphic to S_4. Justify.
10. Find all units and zero divisors of $\mathbb{Z}_{20}$.
11. Show that $2\mathbb{Z} \cup 3\mathbb{Z}$ is not a subring of $\mathbb{Z}$.
12. Find all maximal ideals of $\mathbb{Z}_{12}$.

(2 x 10 =20)

PART B

Answer any 5 (5 marks each)

13. Show that a non empty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H$, for all $a, b \in H$.
14. Let G be a group and a be a fixed element of G. Show that $H_a = \{x \in G | xa = ax\}$ is a subgroup of G, is this subgroup abelian.
15. Construct the group table for D_3, the symmetries of an equilateral triangle.
16. Prove that every permutation on a finite set A is a product of disjoint cycles.
17. Compute the factor group of $\mathbb{Z}_4 \times \mathbb{Z}_6 / < (0,2) >$.
18. Define kernel of a group homomorphism, and show that kernel of a homomorphism is a normal subgroup.

19. Prove that the set of Gaussian integers $\mathbb{Z}[i] = \{a + ib; a, b \in \mathbb{Z}\}$ is a subring of the ring of complex numbers $\mathbb{C}$.
20. Prove Fermat's theorem and using it find the reminder of 3^{47} when divided by 23.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. a) Find all subgroups of $\mathbb{Z}_{18}$ and draw the lattice diagram.

    b) Prove that any cyclic group of infinite order is isomorphic $(\mathbb{Z}, +)$.
22. a) State Lagrange's theorem. Is the converse of Lagrange's theorem is true, justify?

    b) Let H be a subgroup of a group G, prove that the relation $a \equiv_l b (mod\ H)$ if and only if $a^{-1}b \in H; a, b \in H$ is an equivalence relation. Also find the equivalence class.
23. Let H be a subgroup of a group. Show that the left coset multiplication is well defined if and only if every left cosets of H are the same as the right cosets of H.
24. a) Show that a field contains no proper non trivial ideals.

    b) Prove if p is a prime, then $\mathbb{Z}_p$ is a field.

(10 x 3 = 30)

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