22U504-S

Reg. No…………………………………………. Name……………………………………………

B.Sc. DEGREE END SEMESTER EXAMINATION: OCTOBER 2022

SEMESTER **5: MATHEMATICS (CORE COURSE FOR MATHEMATICS AND COMPUTER APPLICATION)**

COURSE: **15U5CRMAT5: MATHEMATICAL ANALYSIS**

(Common for Supplementary 2015/2016/2017/2018 admissions)

Time: Three Hours Max. Marks: 75

PART A

Answer **all** *questions. Each question carries* 1 *mark.*

1. Define open interval
2. Give an example of an open set which is not an interval
3. Find the derived set of the set of all rational numbers.
4. State Cesaro's theorem.
5. Give an example of a sequence which is not monotonic.
6. Find the centre and radius of $|z-1+3i| = 2$
7. Define bounded set with an example.
8. Reduce to a real number: $(1-i)^4$.
9. State Bolzano-Weierstrass Theorem for sets.
10. State Cauchy's General Principle of convergence (1 x 10 = 10)

PART B

Answer ***any*** **eight** *questions. Each question carries* 2 *marks.*

11. Prove that the greatest number of a set if it exists is the supremum of the set.
12. State Dedekind's property of real numbers.
13. Define the derived set of a set S. Obtain the derived set of the open interval (a,b).
14. Prove that every monotonic increasing sequence which is not bounded above diverges to $+\infty$
15. Locate the numbers z_1+z_2 and z_1-z_2 vectorially when $z_2 = 2i$ and $z_2 = 2-i$.
16. Show that the sequence $\{(-1)^n\}$ oscillates finitely.
17. Show that every open set is a union of open intervals.
18. State Archimedean property of real numbers.
19. Sketch the set $|2z + 5| > 3$ where z is a complex number.
20. Prove: Closure of a bounded set is bounded. (2 x 8 = 16)

PART C

Answer ***any*** **five** *questions. Each question carries* 5 *marks.*

21. Prove that infimum of a bounded set is always a member of its closure
22. Prove that the set of real numbers in (0,1) is uncountable .
23. Show that the set of rational numbers in the closed interval [0,1] is countable.
24. Prove that a non-empty finite set is not a neighbourhood of any point

25. State and prove the nested intervals theorem.
26. Prove: A set is closed if and only if its complement is open.
27. Prove: The union of two closed sets is closed. (5 x 5 = 25)

PART D

Answer ***any*** **two** *questions. Each question carries* 12 *marks.*

28. Prove the equivalence of Dedekind's property and order completeness property of real numbers
29. State and prove Cauchy's general principle of convergence.
30. (a) Find the two square roots of $\sqrt{3}+i$.

    (b) Find the principal argument of $\frac{i}{-2-2i}$ and $(\sqrt{3}-i)6$.

31. State the two forms of Completeness Property of real numbers and prove their equivalence.

(12 x 2 = 24)

****