Reg. No .... Name **22U504**

B Sc DEGREE END SEMESTER EXAMINATION - OCTOBER 2022

SEMESTER 5 : MATHEMATICS

COURSE : **19U5CRMAT05 ; REAL ANALYSIS - I**

(For Regular - 2020 Admission and Supplementary - 2019 Admission))

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Prove that $\lim\limits_{x\to 0}\log|x| = -\infty$.
2. Prove that $\lim\limits_{n\to\infty}\dfrac{1}{n}[1+2^{\frac{1}{2}}+3^{\frac{1}{3}}+\ldots+n^{\frac{1}{n}}]=1$.
3. Find the infimum and supremum of the set $\{-2,-\frac{3}{2},-\frac{4}{3},-\frac{5}{4},\ldots,-\frac{n+1}{n},\ldots\}$. Which of these belongs to the set?
4. Show that the series $\sum\sqrt{\dfrac{n}{2(n+1)}}$ is divergent.
5. Define a strictly increasing sequence and give an example.
6. Test for convergence the series $\sum\dfrac{\sqrt{n}}{n^2+1}$
7. Give an example each of (a) a convergent sequence (b) a divergent sequence.
8. Define a conditionally convergent series and give an example.
9. Prove that the greatest member of a set, if it exists, is the supremum of the set.
10. Prove that $|-x|=|x|$.
11. State the order completeness property.
12. Define a monotonic increasing sequence and give an example.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Prove that the positive term geometric series $\sum\limits_{n=0}^{\infty} r^n$, converges for $r<1$ and diverges to ∞ for $r\geq 1$.
14. If S and T are subsets of real numbers, prove that $(S\cup T)'=S'\cup T'$.
15. State and prove Sandwich Theorem.
16. Test for convergence the series $1+\dfrac{1}{2}x+\dfrac{1.3}{2.4}x^2+\dfrac{1.3.5}{2.4.6}x^3+\cdots$
17. Show that the intersection of a finite collection of open sets is open. Is this theorem valid for an arbitrary family of open sets? Justify.
18. Show that $\lim\limits_{x\to 0}\dfrac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}$ does not exist..
19. Show that every convergent sequence is bounded. Is the converse true? Justify.
20. Test for convergence the series $\sum\dfrac{n}{n^2+1}x^n$, where $x>0$.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. State and prove Raabe's test.
22. (a) If $f : A \to B$ is one to one and B is countable, then prove that A is countable.
    (b) Show that every subset of a countable set is countable.
23. State and prove Leibnitz's test for convergence of an alternating series.
24. State and prove the nested interval theorem.

(10 x 3 = 30)