Reg. No .... Name **23U253**

B. Sc. DEGREE END SEMESTER EXAMINATION : MARCH 2023

EMESTER 2 : COMPLEMENTARY MATHEMATICS FOR PHYSICS AND CHEMISTRY

COURSE : **19U2CPMAT2 : CALCULUS – 2 AND NUMERICAL ANALYSIS**

(For Regular - 2022 Admission and Improvement / Supplementary – 2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Find the divergence and curl of the vector $\overrightarrow{V} = \left(xyz\right)i + \left(3x^2y\right)j + \left(xz^2 - y^2z\right)k$ at the point $(2, -1, 1)$
2. What is the Lagrange's formula for unequal intervals ?
3. If $\overrightarrow{r} = xi + yj + zk$, show that $\nabla\left(\overrightarrow{a} \,.\, \overrightarrow{r}\right) = \overrightarrow{a}$, where $\overrightarrow{a}$ is a constant vector
4. Define Simpson's one third rule.
5. State the Gauss Divergence Theorem.
6. Show that $\nabla \times \left(\overrightarrow{A} + \overrightarrow{B}\right) = \nabla \times \overrightarrow{A} + \nabla \times \overrightarrow{B}$
7. If $\overrightarrow{r} = xi + yj + zk$, then evaluate $\left(i\right)\ div\,\overrightarrow{r} \quad \left(ii\right)\ curl\ \overrightarrow{r}$
8. Write the iteration formula for the Regula-Falsi method.
9. Prove that $\Delta \frac{u_x}{v_x} = \frac{v_x \Delta u_x - u_x \Delta v_x}{v_x v_{x+1}}$
10. What is the general form of third approximtion x_1 if a and b are the first two approximations using regula falsi method.
11. Write the rate of convergence of Newton-Raphson method.
12. Show that $\Delta c f(x) = c \Delta f(x)$.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Draw the graph of $y = e^{x-1}$ and find graphically the value of root of the equation $3 - x = e^{x-1}$.
14. Use Green's theorem in the plane to evaluate the integral $\oint_C (2x^2 - y^2)\,dx + (x^2 + y^2)\,dy$ where C is the boundary in the xy- plane of the area enclosed by the x- axis and the semicircle $x^2 + y^2 = 1$ in the upper half xy - plane.
15. Evaluate $\iint_S (y^2z^2\,\hat{i} + z^2x^2\,\hat{j} + z^2y^2\hat{k}).\,\hat{n}\,dS$, where S is the part of the sphere $x^2 + y^2 + z^2 = 1$ above the xy-plane and bounded by this plane.
16. Prove that $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{1}{4}\delta^2}$
17. If $\overrightarrow{a}$ is a constant vector and $\overrightarrow{r} = xi + yj + zk$, prove that $curl\left(\overrightarrow{a} \times \overrightarrow{r}\right) = 2\,\overrightarrow{a}$
18. Find by iteration method, a real root of $2x - \log_{10} x = 7$.

19. Prove that $\Delta log f(x) = log[1 + \frac{\Delta f(x)}{f(x)}]$

20. Find the angle between the tangent planes to the surfaces $x \log\ z = y^2 - 1$ and $x^2 y = 2 - z$ at the point (1,1,1)

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Verify Stoke's theorem for $\overrightarrow{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy- plane.

22. Find a root of the equation $x^3 - x - 11 = 0$, correct to 4 decimals using bisection method.

23. a. Prove that the vector $f\left(r\right)\overrightarrow{r}$ is irrotational

b. Prove that $\nabla^2 f\left(r\right) = f''\left(r\right) + \frac{2}{r} f'\left(r\right)$

24. Express the function $f(x) = 2x^3 + 3x^2 - 5x + 4$ and its successive differences in factorial notation. Also obtain a function whose first difference is $f(x)$.

(10 x 3 = 30)