

**B. Sc. DEGREE END SEMESTER EXAMINATION : MARCH 2023****SEMESTER 2 : MATHEMATICS****COURSE : 19U2CRMAT2: ADVANCED CALCULUS AND TRIGONOMETRY***(For Regular - 2022 Admission and Improvement / Supplementary – 2021/2020/2019 Admissions)*

Time : Three Hours

Max. Marks: 75

**PART A****Answer any 10 (2 marks each)**

1. Assuming you have a triangle with the following points, (0, 0) (5,5) (5,0). What is the integral for the area of this triangle expressed as polar coordinates using a single integral ?
2. If  $y = a \cos(\log x) + b \sin(\log x)$ , show that  $x^2 y'' + xy' + y = 0$ .
3. State Fubini's Theorem.
4. State True or False : The limit of the Reimann sum is the double integral of  $f(x, y)$  over R.
5. Identify the curve by transforming to polar coordinates  $r = 4 \cos \theta + 6 \sin \theta$ .
6. Prove that  $\cos ix = i \cos hx$
7. Find the asymptotes parallel to the coordinate axes of the curve  $y^2 x - a^2(x - a) = 0$ .
8. Evaluate  $\int_0^1 xy^2 dx$
9. Find  $y'(x)$ ,  $y''(x)$  without eliminating the parameter for the curve  $x = \sec t$ ,  $y = \tan t$ ;  $t = \frac{\pi}{4}$ .
10. Express the given equation in polar coordinates a)  $9xy = 4$  .  
b)  $x^2 + y^2 + 8y = 0$
11. If  $\tan \frac{\theta}{2} = \tanh \frac{u}{2}$  then  $\sinh u = \tan \theta$ .
12. Find the asymptote parallel to the coordinate axes for  $x^2 y^2 - a^2(x^2 + y^2) = 0$ .  
**(2 x 10 = 20)**

**PART B****Answer any 5 (5 marks each)**

13. Find the area of the region enclosed by the rose  $r = 2 \sin 2\theta$ .
14. Find the  $n^{\text{th}}$  derivative of  $\frac{x^2}{(x+2)(2x+3)}$ .
15. Show that  $\cos h^{-1} x = \log \left[ x + \sqrt{x^2 - 1} \right]$ , when x is real.
16. Find the nth derivative of  $y = \cos^4 x$ .
17. Find the series  $\sin \alpha + c \sin(\alpha + \beta) + \frac{c^2}{2!} \sin(\alpha + 2\beta) + \dots \infty$
18. Use cylindrical coordinates to evaluate  $\int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_0^{9-x^2-y^2} x^2 dz dy dx$ .
19. Find all values of t at which the parametric curve has a horizontal tangent line and a vertical tangent line where  $x = 2t^3 - 15t^2 + 24t + 7$ ,  $y = t^2 + 4t + 1$ .
20. Use double integration to find the area enclosed by the curves  $y = \sin x$  and  $y = \cos x$  for  $\pi/4 \leq x \leq \pi/2$   
**(5 x 5 = 25)**

**PART C**

**Answer any 3 (10 marks each)**

21. Sum the series  $\cos \alpha + nC_1 \cos(\alpha + \beta) + nC_2 \cos(\alpha + 2\beta) + \dots + \cos(\alpha + n\beta)$
22. Use triple integral to find the volume of the solid cylinder  $x^2 + y^2 = 9$  and between the plane  $z = 1$  and  $x + z = 5$
23. Find the value of the  $n$ th derivative of  $y = e^{m \sin^{-1} x}$  for  $x = 0$ .
24. The sphere of radius  $a$  centered at the origin is expressed in rectangular coordinates as  $x^2 + y^2 + z^2 = a^2$ , and hence its equation in cylindrical coordinates is  $r^2 + z^2 = a^2$ . Use this equation and a polar double integral to find the volume of the sphere.

**(10 x 3 = 30)**