Reg. No .... Name **22U156**

B. Sc. DEGREE END SEMESTER EXAMINATION : OCTOBER 2022

SEMESTER 1 : COMPLEMENTARY MATHEMATICS FOR B. Sc. PHYSICS/CHEMISTRY

COURSE : **19U1CPMAT1 : CALCULUS-1**

(For Regular – 2022 Admission and Improvement / Supplementary - 2021/2020/2019 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Calculate $\int\limits_R \frac{\sin x}{x} dA$ where R is the triangle in the xy- plane bounded by the x-axis, the line $y = x$, and the line $x = 1$.
2. Evaluate f_x and f_y if $f(x, y) = 2x^2 - 3y - 4$.
3. Calculate $\int\int f(x, y)dA$ for $f(x, y) = 1 - 6x^2y$ and $R : 0 \leq x \leq 2,\ -1 \leq y \leq 1$.
4. Verify mean value theorem for the function $f(x) = \sqrt{x-1}$, in the interval $[1, 3]$
5. State first derivative test for local extrema.
6. Let f be continuous on the symmetric interval $[-a, a]$. Show that $\int\limits_{-a}^{a} f(x)dx = 2\int\limits_{0}^{a} f(x)dx$, if f is even.
7. State the stronger form of Fubini's theorem.
8. Find the absolute extrema values of $g(t) = 8t - t^4$ on $[-2, 1]$.
9. Draw a tree diagram and write a Chain Rule formula for $\frac{dz}{dt}$ for $z = f(u, v, w), u = g(t), v = h(t), w = k(t)$.
10. Define volume of a solid.
11. Find the volumes of the solid generated by revolving the region bounded by the lines $y = x,\ y = 1$ and $x = 0$.
12. Evaluate f_x and f_y if $f(x, y) = \cos^2(3x - y^2)$.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Use mean value theorem to show that the functions with zero derivatives are constant.
14. Find the extreme values of $V(x) = x(10 - 2x)(16 - 2x), 0 < x < 5$.
15. A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at a 45° angle at the center of the cylinder. Find the volume of the wedge.
16. Find all the local maxima, local minima, and saddle points of the function $f(x, y) = x^2 + xy + 3x + 2y + 5$.
17. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

18. Evaluate the integral $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln\left(x^2+y^2+1\right) dx\, dy$ by converting the given integral into an equivalent polar integral.

19. Show that the function $w=\cos\left(2x+2ct\right)$ is a solution of the one dimensional wave equation $\frac{\partial^2 w}{\partial t^2}=c^2\frac{\partial^2 w}{\partial x^2}$.

20. Sketch the region of integration and evaluate the integral $\int_0^3\int_{-2}^{0}(x^2y-2xy)dy\ dx$.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Find the area of the "triangular" region in the first quadrant bounded on the left by the y-axis and on the right by the curves $y=\sin x$ and $y=\cos x$.

22. Find the critical points of $f(x)=x^2\sqrt{5-x}$ and identify the intervals on which f is increasing and decreasing. Find the function's local and absolute extreme values.

23. Find the absolute maxima and minima of the function $f(x,y)=4x-8xy+2y+1$ on the triangular plate bounded by the lines $x=0, y=0, x+y=1$ in the first quadrant.

24. A thin plate covers the triangular region bounded by the x - axis and the lines $x=1$ and $y=2x$ in the first quadrant. The plate's density at the point (x,y) is $\delta(x,y)=6x+6y+6$. Find the plate's mass, first moments, and center of mass about the coordinate axes. Also find the moments of inertia and radii of gyration about the coordinate axes and the origin.

(10 x 3 = 30)