

B. Sc. DEGREE END SEMESTER EXAMINATION : OCTOBER 2022**SEMESTER 1 : MATHEMATICS****COURSE : 19U1CRMAT1 : CALCULUS***(For Regular – 2022 Admission and Improvement / Supplementary - 2021/2020/2019 Admissions)*

Time : Three Hours

Max. Marks: 75

PART A**Answer any 10 (2 marks each)**

1. Find the area of the surface generated by revolving the curve $x = 9y + 1, 0 \leq y \leq 2$ about y axis.
2. Find the area of the region enclosed by the curves $y = x^2, y = \sqrt{x}, x = \frac{1}{4}$ and $x = 1$.
3. Evaluate $\lim_{x \rightarrow +\infty} \frac{x^{-4/3}}{\sin\left(\frac{1}{x}\right)}$
4. Define a stationary point.
5. Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x^2}$.
6. Prove that a general cubic polynomial $f(x) = ax^3 + bx^2 + cx + d, (a \neq 0)$ has exactly one inflection point.
7. Find the equation of the level curve that passes through the point (-1,1) Where $f(x, y) = ye^x$.
8. Find f_x and f_y for $f(x, y) = 2x^2y^3 + 2y + 4x$.
9. Find the area of the region bounded above by $y = x + 6$ and below by $y = x^2$ and bounded on the sides by the lines $x = 0, x = 2$
10. Find $\lim_{x \rightarrow \frac{\pi}{4}} (1 - \tan x) \sec 2x$.
11. Find all critical points of $f(x) = 3x^{5/3} - 15x^{2/3}$.
12. Describe the domain of $f(x, y, z) = e^{xyz}$.

(2 x 10 = 20)**PART B****Answer any 5 (5 marks each)**

13. Show that among all rectangles with perimeter p, the square has the maximum perimeter.
14. Verify whether Mean value theorem holds and if so find C where $f(x) = x - \frac{1}{x}; [3, 4]$
15. Find the relative extrema of $f(x) = x^4 - 12x^3$.
16. If $w = e^{xyz}, x = 3u + v, y = 3u - v, z = u^2v$, use chain rule to find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$.
17. Show that the function $u(x, t) = \sin(x - ct)$ is a solution of $\frac{\partial^2 u}{\partial t^2} = c^2 \cdot \frac{\partial^2 u}{\partial x^2}$.
18. Find the area of the surface generated by revolving the curve $x = y^3, 0 \leq y \leq 1$ about y-axis.

19. Use cylindrical shells to find the volume of the solid generated when the region enclosed by the given curves is revolved about x-axis $y^2 = x, y = 1, x = 0$.
20. Verify that the hypotheses of the Rolle's theorem are satisfied on the given interval and find all values of c in that interval that satisfy the conclusion of the theorem. $f(x) = \cos x, \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Determine the volume of the solid that results when the region enclosed by $x = y^2$ and $x = y$ is revolved about the line $y = 1$.
22. Evaluate $\lim_{x \rightarrow 0} \frac{1}{x^2} - \frac{1}{\sin^2 x}$ and $\lim_{x \rightarrow 0^+} \sin x \log x$.
23. Sketch the graph of curve $y = \frac{(x-1)(x-3)}{x^2}$.
24. Find the points on the sphere $x^2 + y^2 + z^2 = 36$ that is closest to and farthest from the point $(1, 2, 2)$.

(10 x 3 = 30)