Reg. No .... Name **23P4027-S**

M. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2023

SEMESTER 4 : MATHEMATICS

COURSE : **16P4MATT18EL: COMBINATORICS**

(For Supplementary - 2016/2017/2018/2019/2020 Admissions)

Time : Three Hours Max. Marks: 75

PART A
Answer All (1.5 marks each)

1. Prove that the number of r-permutations of the set A ={a_1,a_2,......a_n} where r, $n\epsilon\ N$, with repetitions allowed, is given by n^r
2. Show that (4n)! is a multiple of 2^{3n} . 3^n , for each natural number n.
3. A 4-storey house is to be painted by some 6 different colours such that each storey is painted in one colour. How many ways are there to paint the house?
4. Prove that among any group of 13 people, there must be at least 2 whose birthdays are in the same month.?
5. State the Generalized Pigeonhole Principle.
6. Derive the formula for $|A \cup B \cup C|$
7. State Fundamental Theorem of Arithmetic?
8. State the problem of Tower of Hanoi?.
9. Find the exponential generating function for (0!, 1!, 2!, ... , r!, ...)
10. Find the generating functions for the sequence (c_r), where $(c_r) = (a_0, a_1 - a_0, a_2 - a_1 \dots).$

(1.5 x 10 = 15)

PART B
Answer any 4 (5 marks each)

11. In how many ways can a committee of 5 be formed from a group of 11 people consisting of 4 teachers and 7 students if
(i) there is no restriction in the selection?
(ii) the committee must include exactly 2 teachers?
(iii) the committee must include at least 3 teachers?
(iv) a particular teacher and a particular student cannot be both in the committee?
12. Between 20000 ad 70000,find the number of even integers in which no digit is repeated.
13. Prove that at a gathering of any six people. Some three of them are either mutual acquaintances or complete strangers to one another.
14. Explain and find the formula for Stirling number of the 2nd kind?
15. Find the number of ways to select 4 members from the mu I ti-set M = {2 . b, 1 . c, 2 . d, 1 . e}.
16. For each$r \epsilon N^*$, find a_r, the number of ways of distributing r distinct objects into n distinct boxes such that no box is empty.

(5 x 4 = 20)

PART C

Answer any 4 (10 marks each)

17.1. Let S be the set of natural number whose digits are chosen from {1,3,5,7} such that no digits are repeated. Find

(1). |S|;

(2). $\sum_{n \epsilon S} n$

OR

2. a) In a group of 15 students, 5 of them are female. If exactly 3 female students are to be selected, in how many ways can 9 students be chosen from the group

(i) to form a committee?

(ii) to take up 9 different posts in a committee?

b) Ten chairs have been arranged in a row. Seven students are to be seated in seven of them so that no two students share a common chair. Find the number of ways this can be done if no two empty chairs are adjacent.

c) Let $r \epsilon N$, such that

$$\frac{1}{\binom{9}{r}} - \frac{1}{\binom{10}{r}} = \frac{11}{6\binom{11}{r}}.$$

Find the value of r ?

18.1. Define Ramsey number and show that R(3,3,) = 6.

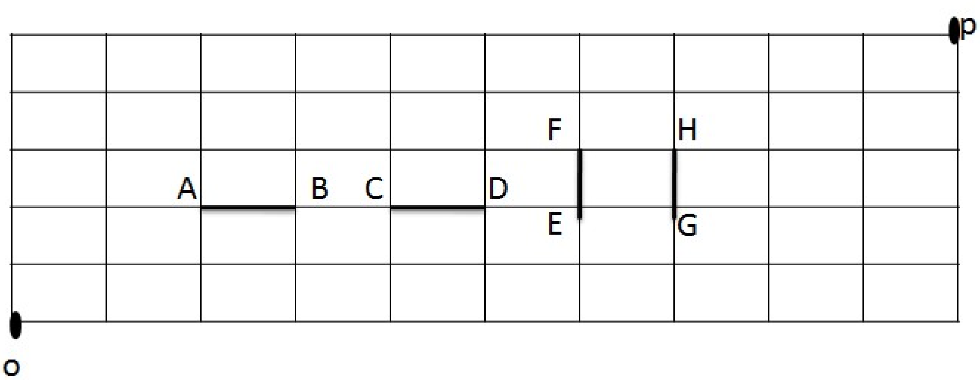
OR

2. a) Ten players took part in a round robin chess tournament (i.e., each player must play exactly one game against every other player). According to the rules, a player scores 1 point if he wins a game; -1 point if he loses; and 0 point if the game ends in a draw. When the tournament was over, it was found that more than 70% of the games ended in a draw. Show that there were two players who had the same total score.?

b) Let A = {a_1, a_2, ... , a_5} be a set of 5 positive integers. Show that for any permutation $a_{i1}, a_{i2}, a_{i3}, a_{i4}, a_{i5}$ of A, the product.

$(a_{i1} - a_1)(a_{i2} - a_2)\ldots\ldots\ldots(a_{i5} - a_5)$ is always even.

19.1.



The figure showes a 11 by 6 rectangular grid with 4 specified segments A B, CD, E F and G H. Find the number of shortest routes from O to P in each of the following cases using the method of properties:

(i) All the 4 segments are deleted;

(ii) Each shortest route must pass through exactly 2 of the 4 segments.

OR

2. Let $n\epsilon N$,and let $n = p_1^{m_1} p_2^{m_2} \ldots p_k^{m_k}$, be its prime fractorization then show that $\varphi(n) = n \prod_{i=1}^{k} \left(1 - \frac{1}{p_i}\right)$.

20.1. Explain Distribution problems using generating functions

OR

2. a) Show that the generating function for the number of ways to select r objects from 3 distinct objects is $(1+x)^3$.

b) Express the generating function for each of the following sequences (c_r) in closed form (i.e., a form not involving any series):

i) $c_r = 3r + 5 \ for\ each\ r \ \epsilon\ N^*$;.

$ii) c_r = r^2 \ for\ each\ r\ \epsilon\ N^*$;

(10 x 4 = 40)