Reg. No .................... Name .............................. **23P4027**

**M. Sc. DEGREE END SEMESTER EXAMINATION : MARCH 2023**

**SEMESTER 4 : MATHEMATICS**

COURSE : **21P4MATTEL18 : PROBABILITY THEORY**

*(For Regular - 2021 Admission)*

Duration : Three Hours Max. Weights: 30

**PART A**

**Answer any 8 questions** **Weight: 1**

1. Define a null event and a certain event. (R, CO 1)
2. Find the probability generating function for rv $X$ with pmf $P(X=k)=pq^k, k=0,1,2,\ldots,0<p<1, q=1-p$. (A, CO 2)
3. Let $X_1, X_2, \ldots$ be iid Poisson rv's with parameter $\lambda$. Use CLT to estimate $P(120 \leq S_n \leq 160)$ where $S_n = X_1+X_2+\ldots+X_n, \lambda=2, n=75$ and $P(-2.45 \leq Z \leq 0.82)=0.7868$. (A, CO 4)
4. Define probability measure. (R, CO 1)
5. Let $(X,Y)$ be jointly distributed with pdf $f(x,y)=2, 0<x<y<1$, and $=0, otherwise$.Find the marginal pdf's. (A, CO 3)
6. Let X be a Poisson rv with parameter $\lambda$. Find its truncated pmf if $T$ ={$X \geq 1$}. (A, CO 3)
7. Define conditional expection for discrete and continuous case. (U, CO 3)
8. Let $F_n$ be a sequnce of distribution functions defined by $F_n(x)=0, x<0; 1-\frac{1}{n}; 1, x \geq n$.Check convergence in law. (A, CO 4)
9. Let $X$ be the rv defined on a probability space $(\Omega, S, P)$ by $X(\omega)=c, \forall \omega \in \Omega$. Find the DF of $X$. (U, CO 2)
10. Does $F(x)=1-e^{-x}, x \geq 0$ and $=0, x<0$ define a DF? (A, CO 2)

**(1 x 8 = 8)**

**PART B**

**Answer any 6 questions** **Weights: 2**

11. Find the quartile of order $p(0<p<1)$ for rv $X$ with pdf $f(x)=\begin{cases} \frac{1}{x^2}, x \geq 1 \\ 0, otherwise \end{cases}$. (A, CO 2)
12. Show that two rv's $X$ and $Y$ are independent iff for every pair of Borel measurable functions $g_1$ and $g_2$ the relation $E(g_1(X)g_2(Y)) = E(g_1(X))E(g_2(Y))$ holds, provided that the expectations on both sides of the equation exists. (An, CO 3)
13. Define PGF. Consider the PGF, $P(s)=(\frac{p}{1-sq})$. Then find the mean and variance of rv $X$. (A, CO 2)
14. Let $A, B, C$ be three independent events. Show that $A^c, B^c$ and $C^c$ are independent. (A, CO 1)

15. Let $(X_1, X_2, X_3)$ be a rv with jpdf $f(x_1, x_2, x_3) = \begin{cases} \frac{1}{4}, (x_1, x_2, x_3) \in A \\ 0, otherwise \end{cases}$ (A, CO 3)

where
$A =\{(1,0,0), (0,1,0), (0,0,1), (1,1,1)\}$.Are $X_1, X_2, X_3$ independent? Are they pairwise independent?

16. Let $\{X_n\}$ be a sequence of random variables with pmf $P(X_n = 0) = 1 - \frac{1}{n^r}$ and $P(X_n = n) = \frac{1}{n^r}, r > 0, n = 1, 2, 3, \ldots$.Show that $X_n \overset{r}{\nrightarrow} 0$ but $X_n \xrightarrow{P} 0$. (A, CO 4)

17. Two fair dice are thrown independently.Let $A$ denote the event odd number on first dice, $B$ denote the event odd number on second dice and $C$denote the event that sum of numbers on both dice is odd. Are $A, B, C$ mutually independent ? (An, CO 1)

18. Let $X_1, X_2, \ldots, X_n$ be iid random variables with commomn density function $f(x) = \frac{1}{\theta}, 0 < x < \theta; 0, otherwise$ , where $\theta > 0$.Let $X_{(n)} = max\{X_1, X_2, \ldots, X_n\}$.Does $F_n$ converge in law? (A, CO 4)

**(2 x 6 = 12)**

**PART C**

**Answer any 2 questions** **Weights: 5**

19. Show that convergence in probability implies convergence in distribution.When does convergence in distribution imply convergence in probability ? Justify? (An, CO 4)

20. State Bayes theorem.Each of $n$ urns contains four white and six black balls, while another urn contains five white and five black balls. An urn is chosen at random from the $n + 1$ urns, and two balls are drawn from it, both being black. The probability that five white and three black balls remain in the chosen urn is $\frac{1}{7}$. Find $n$. (An, CO 1)

21. Define Distribution function.Show that the set of discontinuity points of a DF is at most countable. (A, CO 2)

22. Define conditional expectation for discrete and continuous rv's. If $(X, Y)$ has joint pmf $p(0,0) = \frac{4}{25}, p(0,1) = \frac{6}{25}, p(1,0) = \frac{6}{25}, p(1,1) = \frac{9}{25}$ then find $E(X|Y)$ and $E(Y|X)$. (A, CO 3)

**(5 x 2 = 10)**

OBE: Questions to Course Outcome Mapping

| CO | Course Outcome Description | CL | Questions | Total Wt. |
|---|---|---|---|---|
| CO 1 | define the principal concepts about probability and evaluating it. | A | 1, 4, 14, 17, 20 | 11 |
| CO 2 | explain the concept of a random variable and the probability distributions | A | 2, 9, 10, 11, 13, 21 | 12 |
| CO 3 | Analyze the concept of function of rv's and multiple rv's | An | 5, 6, 7, 12, 15, 22 | 12 |
| CO 4 | Analyze the concept of convergence of sequence of rv's | An | 3, 8, 16, 18, 19 | 11 |

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;