Reg. No .... Name **22P354**

M. Sc. DEGREE END SEMESTER EXAMINATION : OCTOBER 2022

SEMESTER 3 : MATHEMATICS

COURSE : **21P3MATT15 : MULTIVARIATE CALCULUS**

(For Regular - 2021 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Explain the term Jacobian matrix (A, CO 2)
2. Let f be a function with values in $\mathbb{R}^m$ which is differentiable at a point c in $\mathbb{R}^n$ with total derivative $f'(c)$. Show that $||f'(c)(v)|| \leq M||v||$, where $M = \sum_{k=1}^{m} ||\nabla f_k(c)||$. (A, CO 2)
3. State Fourier Integral theorem (A, CO 1)
4. Explain k - surface (U, CO 4)
5. Define basic k-forms (U, CO 4)
6. Prove that $f * g = g * f$. (A, CO 1)
7. State the exponential form of Fourier Integral Theorem (A, CO 1)
8. State the second derivative test for extrema (U, CO 3)
9. Show that $f_\omega = xdy + ydx$ then $\int_\gamma \omega = 0$ for every 1- surface closed curve γ. (A)
10. Let $f : \mathbb{R} \to \mathbb{R}^2$ be a function given by $f(t) = (\cos t, \sin t)$. Show that the ordinary Mean Value theorem does not hold in $[0, 2\pi]$. (An, CO 3)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ where Q is the point $(5, 0, 4)$ (A, CO 2)
12. If f is differentiable at c, then prove that f is continuous at c. (A, CO 2)
13. Find the points on the surface $z^2 = xy + 1$ nearest to the origin (An, CO 3)
14. If f satisfies the hypotheses of the Fourier integral theorem show that
(a) If f is even then
$\frac{f(x+)+f(x-)}{2} = \frac{2}{\pi} \lim_{\alpha\to\infty} \int_0^\alpha \cos vx \left[\int_0^\infty f(u) \cos vu \, du\right] dv.$ (A, CO 1)
(b) If f is odd then $\frac{f(x+)+f(x-)}{2} = \frac{2}{\pi} \lim_{\alpha\to\infty} \int_0^\alpha \sin vx \left[\int_0^\infty f(u) \sin vu \, du\right] dv.$
15. Suppose T is a C' mapping of an open set $E \subset R^n$ into an open set $V \subset R^m$, S is a C'- mapping of V into an open set $W \subset R^p$, and ω is a k- form in W, so that ω_S is a k-form in V and both $(\omega_S)_T$ prove that ω_{ST} are k-forms in E, where ST is defined by $(ST)(X) = S(T(X))$. Then $(\omega_S)_T = \omega_{ST}$ (An, CO 4)
16. Let D be the 3- cell defined by $0 \leq r \leq 1, 0 \leq \theta \leq \pi, 0 \leq \phi \leq 2\pi$ and $\Phi(r, \theta, \phi) = (x, y, z)$ where $x = r \sin\theta \cos\phi, y = r \sin\theta \sin\phi$ and (An, CO 4)

$z = r\cos\theta$. Show that $\int_{\Phi} dx \wedge dy \wedge dz = \frac{4\pi}{3}$.

17. Show that convolution may not be defined if f and g are Lebesque integrable (A, CO 1)

18. For some integer $n \geq 1$, let f have a continuous n^{th} derivative in the open interval (a, b). Suppose also that for some interior point c in (a, b), we have, $f'(c) = f''(c) = \cdots = f^{(n-1)}(c) = 0$, but $f^n(c) \neq 0$. Prove that for n even, f has a local minimum at c if $f^n(c) > 0$, and a local maximum at c if $f^n(c) < 0$. If n is odd, there is neither a local maximum nor a local minimum at c. (A, CO 3)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Assume that g is differentiable at a, with total derivative $g'(a)$. Let $b = g(a)$ and assume that f is differentiable at b, with total derivative $f'(b)$. Then prove that the composite function $h = fog$ is differentiable at a, and the total derivative $h'(a)$ is given by $h'(a) = f'(b)og'(a)$, the composition of the linear functions $f'(b)$ and $g'(a)$. (A, CO 2)

20. Suppose ω is k-form in an open set $E \subset R^n$, ϕ is a surface in E, with parameter domain $D \subset R^k$ and Δ is the k- surface in R^k with parameter domain D, defined by $\Delta(u) = u(u \in D)$ then show that $\int_{\phi} \omega = \int_{\Delta} \omega_{\phi}$. (An, CO 4)

21. State and prove Fourier Integral theorem (A, CO 1)

22. (a) State and prove second derivative test for extrema.
(b)Find and classify the extremum values of the function
f(x, y) = $x^2 + y^2 + x + y + xy$. (A)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Explain Weirstras theorem, otherforms of Fourierseries, the Fourier integral theorem, the exponential form of the Fourier integral theorem, integral transforms and convolutions, the convolution theorem for Fourier transforms.	A	3, 6, 7, 14, 17, 21	12
CO 2	Analyze Multivariable Differential Calculus The directional derivative, directional derivatives and continuity, the total derivative, the total derivative expressed in terms of partial derivatives, An application of the complex- valued functions, the matrix of a linear function, the Jacobian matrix, the chain rate matrix form of the chain rule.	A	1, 2, 11, 12, 19	11
CO 3	Interpret Implicit functions and extremum problems, the mean value theorem for differentiable functions, a a sufficient condition for differentiability.	An	8, 10, 13, 18	6
CO 4	Explain the Integration of Differential Forms, primitive mappings, partitions of unity, change of variables, differential forms, and Stoke's theorem.	An	4, 5, 15, 16, 20	11

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;