Reg. No .... Name **22P340**

M. Sc. DEGREE END SEMESTER EXAMINATION : OCTOBER 2022

SEMESTER 3 : MATHEMATICS

COURSE : **21P3MATT14 : ADVANCED COMPLEX ANALYSIS**

(For Regular - 2021 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1.	If $Rez_n \geq 0$ then prove that the product $\prod_{k=1}^{n} z_n$ converges absolutely iff the series $\sum_{n=1}^{\infty}(z_n - 1)$ converges absolutely.	(An)
2.	Let G be an open connected subset of C. For any f in $H(G)$ there is a sequence of polynomials that converges to f in $H(G)$ then show that for any f in $H(G)$ and any closed rectifiable curve γ in G, $\int_{\gamma} f = 0$	(A, CO 2)
3.	State Mittag-Leffler's theorem	(R)
4.	If $\|z\| \geq \frac{1}{2}$ then prove that $\frac{1}{2} \leq \|log(1+z)\| \leq \frac{3}{2}\|z\|$	(An, CO 1)
5.	If $\mathbf{f(z)}$ and $\mathbf{f(\bar{z})}$ are analytic in a region D, then show that $\mathbf{f(z)}$ is constant in that region D.	(An, CO 3)
6.	Prove that an analytic function with constant imaginary part is constant	(An, CO 3)
7.	State Jensen's Formula	(R, CO 4)
8.	State Hadamard factorization theorem	(R)
9.	If for any f in $H(G)$ such that $f(z) \neq 0$ for all z in G there is a function g in$H(G)$ such that $f(z) = [g(z)]^2$. then show that G is homeomorphic to the unit disk, where G is an open connected subset of C.	(A)
10.	State Bohr- Mollerup theorem by defining the Gamma function	(An)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11.	State and prove Euler'stheorem	(U)
12.	Let G be a region and suppose that u is a continuous real valued function on G with the MVP. If there is a point a in G such that $u(a) \geq u(z)$ for all z in G then show that u is a constant function.	(R, CO 3)
13.	State and prove Jensen's Formula	(U, CO 4)
14.	State and prove the Weierstrass factorization theorem	(U)
15.	Let $\{a_n\}$ be a sequence in $\mathbb{C}$ such that $lim\|a_n\| = \infty$ and $a_n \neq 0$ for all $n \geq 1$. If $\{p_n\}$ ia any sequence of integers such that $\sum_{n=1}^{\infty}(\frac{r}{\|a_n\|})^{p_n+1} \leq \infty$ for all $r \geq 0$ then $f(z) = \prod_{n=1}^{\infty} E_{p_n}(z/a_n)$ converges in $H(\mathbb{C})$. The function f is an entire function with zeros only at the points a_n. If z_0 occurs in the sequence $\{a_n\}$ exactly m times then prove that f has a zero at $z = z_0$	(R)

of multiplicity m. Furthermore, if $p_n = n - 1$ then $\sum_{n=1}^{\infty} (\frac{1}{|a_n|})^{p_n+1} \leq \infty$ will be satisfied.

16. Let G be an open connected subset of C.For any f in $H(G)$ and any closed rectifiable curve γ in G, $\int_{\gamma} f = 0$ then show that every function f in $H(G)$ has a primitive (An)

17. Let G be an open connected subset of C. If for any f in $H(G)$ such that $f(z) \neq 0$ for all z in G there is a function g in$H(G)$ such that $f(z) = [g(z)]^2$, then show that If $u : G \longrightarrow R$ is harmonic then there is a harmonic function $v : G \longrightarrow R$ such that $f = u + iv$ is analytic on G (U)

18. Let $D = \{z : |z| < 1\}$ and suppose that $f : \bar{D} \longrightarrow C$ is a continuous function such that both Ref and Imf are harmonic. Show that $f(re^{i\theta}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(e^{it}) P_r(\theta - t) dt$ for all $re^{i\theta}$in D. (A, CO 3)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Let $\prod_{n=1}^{\infty} X_n, d)$, is a metric space. If $\xi^k = \{x_n^k\}_{n=1}^{\infty}$ is in $\prod_{n=1}^{\infty} X_n$, then prove that $\xi^k \longrightarrow \xi = \{x_n\}$ iff $x_n^k \longrightarrow x_n$ for each n. Also show that if each (X_n, d_n) is compact then X is compact. (R)

20. Let G be an open subset of the plane and let E be a subset of $\mathbb{C}_\infty - G$ such that$E$ meets every component of $\mathbb{C}_\infty - G$. Let $R(G, E)$ be the set of rational functions with poles in$E$and consider $R(G, E)$ as a subspace of $H(G)$. If $f \in H(G)$ then show that there is a sequence $\{R_n\}$ in $R(G, E)$ such that $f = limR_n$. That is, $R(G, E)$ is dense in $H(G)$. (A)

21. State and prove Riemann mapping theorem (U)

22. State and prove Harnack's Inequality. (U, CO 3)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Explain the space of functions, Riemann mapping theorem and Weierstrass factorization theorem.	U	4	1
CO 2	Analyze Runge's Theorem, Simple connectedness, MittagLeffler's theorem, Analytic continuation and Riemann surfaces, Schwartz Reflection Principle, Analytic continuation along a path, Mondromy theorem.	An	2	1
CO 3	Interpret Harmonic functions, Basic properties of harmonic functions and Harmonic functions on the disk.	U	5, 6, 12, 18, 22	11
CO 4	Perceive Entire functions, Jensen's formula, the genus and order of an entire function, Hadamard Factorization theorem.	U	7, 13	3

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;