Reg. No .... Name **22P324**

M. Sc. DEGREE END SEMESTER EXAMINATION : OCTOBER 2022

SEMESTER 3 : MATHEMATICS

COURSE : **21P3MATT13 : ADVANCED TOPOLOGY**

(For Regular - 2021 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. If a space X is Hausdorff prove that limits of all nets in it are unique. ()
2. Prove that the evaluation function of a family of functions $\{f_i\}$ is continuous if each f_i is continuous. ()
3. Define base and sub-base of a filter on a set X. ()
4. State Tietze characterization of normality. ()
5. Let S be a sub-base for a topological space X. Prove that if X is completely regular, then for each $V \in S$ and for each $x \in V$, there exist a continuous function $f : X \to [0,1]$ such that $f(x) = 0$ and $f(y) = 1$ for all $y \notin V$. ()
6. State Urysohn's Lemma. (An)
7. Let $X^+ = XU\{\infty\}$ be the one point compactification of the space X. Prove that if $\{\infty\}$ is open in X^+, then X is compact. ()
8. Prove that a first countable, countably compact space is sequentially compact. (An)
9. Define a filter associated with a net S in X. ()
10. If a space is embeddable in the Hilbert cube, prove that it is second countable and T_3. ()

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Prove that in a second countable space, compactness, countable compactness and sequential compactness are all equivalent to each other and hence these three forms of compactness are equivalent in a metric space. ()
12. Let $f_1, f_2, f_3 : \mathbb{R} \to \mathbb{R}$ be defined by $f_1(x) = \cos x$, $f_2(x) = \sin x$, $f_3(x) = x$ for $x \in \mathbb{R}$. Describe the evaluation maps of the families $\{f_1, f_2\}$; $\{f_1, f_3\}$ and $\{f_1, f_2, f_3\}$. Which of these families distinguish points? ()
13. If a topological space is Tychonoff, prove that it is embeddable into a cube. ()
14. Let A be a closed subset of a normal space X and suppose $f : A \to [-1,1]$ is continuous. Then prove that there exists a continuous function $F : X \to [-1,1]$ such that $F(x) = f(x) \ \forall \ x \in A$. (U)
15. Prove that a space is compact iff every ultra filter in it is convergent. ()

16. Prove that the net associated with an ultra filter is a universal net. ()

17. State Urysohn's characterization of normality and using it prove that a connected, T_4 space with at least two points must be uncountable. ()

18. Prove that the Alexandroff compactification of a non-empty, connected space is connected iff that space is not compact. ()

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. State and prove Urysohn Embedding Theorem. ()

20. State and prove two characterizations of compact spaces. ()

21. Prove that the one-point compactification of a space is Hausdorff iff the space is locally compact and Hausdorff. ()

22. If the product space is non-empty, prove that each co-ordinate space is embeddable in it and hence prove that if a topological product is T_0, T_1, T_2 or regular, then each co-ordinate space has the corresponding property. ()

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;