Reg. No .... Name **22P315**

M. Sc. DEGREE END SEMESTER EXAMINATION : OCTOBER 2022

SEMESTER 3 : MATHEMATICS

COURSE : **21P3MATT12 : FUNCTIONAL ANALYSIS**

(For Regular - 2021 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Let X be a Banach space and $A \in BL(X)$. Define spectral value of A and spectrum of A. Also define the resolvent set of A. (R, CO 3)
2. Write the usual norm on K^n where $K = \mathbb{R}\, or\, \mathbb{C}$. (U, CO 1)
3. Let X be a normed linear space and $P : X \to X$ be a projection. Show that $I - P : X \to X$ is also a projection, where I is the identity map on X. (U, CO 2)
4. Show that the operation of addition on a normed linear space is continuous. (An, CO 1)
5. Let X and Y be Banach spaces and $F : X \to Y$ be linear. If the graph of F is closed in $X \times Y$ and if F is both one-one and onto, show that F is a linear homeomorphism. (A, CO 2)
6. When are two normed linear spaces said to be linearly homeomorphic? When are two normed linear spaces said to be linearly isometric? (U, CO 1)
7. Define finite rank operator. Also define rank of an operator. (U, CO 3)
8. Is the closed unit ball in an infinite dimensional space compact? Justify your answer. (An, CO 1)
9. Let X be a Banach space and $F \in BL(X)$. Show that F^{-1} exists and belongs to $BL(X)$ if and only if F is one-one and onto. (A)
10. Let X be a normed linear space. Explain the canonical imbedding of X in its second dual X''. (U)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. If an operator A is of finite rank on a linear space, show that $A - I$ is one-one if and only if $A - I$ is onto. (An, CO 3)
12. Let $X = K^n$ with $||.||_p$; $1 \leq p \leq \infty$ and Y be any normed linear space. Show that any linear map $F : X \to Y$ is continuous. (A, CO 1)
13. Let X be a normed linear space and Y be a subspace of X. Show that Y and $\overline{Y}$ are normed linear subspaces of X. (A, CO 1)
14. Let X be a normed linear space. If $E_1 \subset X$is open and $E_2 \subset X$, then show that $E_1 + E_2$ is open. (An)
15. Let X and Y be normed linear spaces, $F \in BL(X, Y)$ and F' be the transpose of F. Prove that $||F|| = ||F'||$ and the map $T : BL(X, Y) \to BL(Y', X')$ defined by $T(F) = F'$ is a linear isometry. (An, CO 4)

16. State and prove a sufficient condition for the continuity of linear maps on Banach sequence spaces. (An, CO 2)

17. Show that a Banach space cannot have a denumerable basis. (An, CO 2)

18. Let X be a finite dimensional linear space and $A : X \to X$ be linear . Show that A is one-one if and only if A is onto. (An, CO 3)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Let $X \neq \{0\}$ and Y be normed linear spaces. Show that $BL(X, Y)$ is a Banach space if and only if Y is a Banach space. (E, CO 2)

20. Let X and Y be normed linear spaces and $F : X \to Y$ be a linear map. Show that the following statements are equivalent.
(1) F is bounded in some closed ball about zero of positive radius.
(2) F is continuous at zero.
(3) F is continuous on X.
(4) F is uniformly continuous on X.
(5) $||F(x)|| \leq \alpha||x||$ for all $x \in X$ and some $\alpha > 0$.
In particular if $Y = K$, show that each of the above conditions is equivalent to
(6) The hyperspace $Z(F)$ is closed in X.($F \not\equiv 0$). (E, CO 1)

21. Show that the dual of c_{00} with $||.||_p, 1 \leq p \leq \infty$, is l^q where $\frac{1}{p} + \frac{1}{q} = 1.$ (E, CO 4)

22. Determine $e(S),\ a(S)$ and $s(S)$ where S is the right shift operator on a Banach sequence space X. (E, CO 3)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Analyze normed linear spaces, continuity of linear maps , theory and applications of the Hahn-Banach Theorems	E	2, 4, 6, 8, 12, 13, 20	13
CO 2	Analyze Banach spaces, Uniform boundedness principle and the Closed graph theorem.	E	3, 5, 16, 17, 19	11
CO 3	Analyze the Open Mapping theorem, the eigen spectrum and spectral radius.	E	1, 7, 11, 18, 22	11
CO 4	Analyze duals of a normed linear space and transposes of bounded linear maps.	E	15, 21	7

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;