

**M. Sc DEGREE END SEMESTER EXAMINATION : OCTOBER 2022****SEMESTER 3 : MATHEMATICS****COURSE : 21P3MATT11 : PARTIAL DIFFERENTIAL EQUATIONS***(For Regular - 2021 Admission)*

Duration : Three Hours

Max. Weights: 30

**PART A****Answer any 8 questions****Weight: 1**

1. Find the complete integral of  $p^2 = qz$ . (A)
2. Eliminate the parameters a and b from the equation  $z^2(1 + a^3) = 8(x_1 + ax_2 + b)^3$  to find the corresponding pde. (A)
3. Explain the method of separation of variables. (U)
4. Find the complete integral of  $p + q = pq$ . (A)
5. Solve  $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = \frac{2\partial^4 z}{\partial x^2 \partial y^2}$  (A)
6. Form pde by eliminating arbitrary function from  $F(x_1 x_2 + z^2, x_1 + x_2 + z) = 0$ . (A)
7. Define linear and semi linear pde. (U)
8. Prove that  $F(D, D')e^{ax+by} = F(a, b)e^{ax+by}$  (A)
9. Show that the pdes  $z = px_1 + qx_2$  and  $f(x_1, x_2, z, p, q) = 0$  are compatible if the latter is homogeneous in  $x_1, x_2, z$ . (A)
10. Classify the pde as elliptic, hyperbolic or parabolic  $z_{xx} + z_{yy} = 0$ . (U)

**(1 x 8 = 8)****PART B****Answer any 6 questions****Weights: 2**

11. Solve  $z(x_1 p - x_2 q) = x_2^2 - x_1^2$ . (A)
12. Solve  $x_1(x_2 - z)p + x_2(z - x_1)q = z(x_1 - x_2)$ . (A)
13. Find the complete integral of  $(x_1^2 - x_2^2)pq - x_1 x_2(p^2 - q^2) - 1 = 0$ . (An)
14. Solve  $(D^2 - 2DD' - 15D'^2)z = 12xy$ . (A)
15. Solve  $(q + 1)s = (p + 1)t$  using Monge's method. (A)
16. Solve  $(D^2 - D'^2)z = x - y$ . (A)
17. Find the complete integral of
  - a)  $p^2 + q^2 = x_1 + x_2$  (A)
  - b)  $z - p^2 + q^2 = 0$ .
18. By separating the variables, show that the equation  $z_{xx} = \frac{1}{k} z_t$  has solution of the form  $z(x, t) = \sum_0^\infty c_n \cos(nx + \epsilon_n) e^{-n^2 kt}$ . (A)

**(2 x 6 = 12)**

**PART C**  
**Answer any 2 questions**

**Weights: 5**

19. Prove that a necessary and sufficient condition for a Pfaffian differential equation  $X \cdot dr = 0$  to be integrable is that  $X \cdot \text{curl}X = 0$ . (An)
20. Find a complete integral of  $(p^2 + q^2)x_1 = pz$  and deduce the solution which passes through  $x_1 = 0, z^2 = 4x_2$ . (An)
21. Reduce to canonical form and solve the pde  $r + 2s + t = 0$ . (E)
22. Explain in detail about Neumann problem for a rectangle. (An)
- (5 x 2 = 10)**

**OBE: Questions to Course Outcome Mapping**

CO	Course Outcome Description	CL	Questions	Total Wt.
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Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;