Reg. No .... Name **23P2043**

M. Sc. DEGREE END SEMESTER EXAMINATION : MARCH 2023

SEMESTER 2 : MATHEMATICS

COURSE : **21P2MATT09 : NUMBER THEORY**

(For Regular - 2022 Admission and Supplementary - 2021 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Prove that φ is multiplicative. (A, CO 1)
2. Prove that Dirichlet inverse of μ is u. (A, CO 1)
3. Prove or disprove: Every non-zero arithmetical function has Dirichlet inverse. (A, CO 1)
4. Prove that Congruence is an equivalence relation. (A, CO 2)
5. Solve the congruence $5x \equiv 3 \pmod{24}$. (U, CO 2)
6. Prove that $\hat{a} = \hat{b}$ if and only if, $a \equiv b \pmod{m}$. (U, CO 2)
7. Let D be a domain and x and y non-zero elements of D. Prove that x is a unit if and only if $\langle x \rangle = D$. (A, CO 4)
8. Prove that the ring of integers $\mathfrak{O}$ in a number field K is noetherian. (U, CO 4)
9. Prove that $\mathbb{R}[x, y]/\langle x \rangle$ is isomorphic(as rings) to $\mathbb{R}[y]$. (A, CO 4)
10. If $\mathfrak{a} \neq 0$ is an ideal of $\mathfrak{O}$ with $\mathrm{N}(\mathfrak{a})$ is prime , prove that $\mathfrak{a}|\mathrm{N}(\mathfrak{a})$ (A, CO 5)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Find all integers n such that $\varphi(n) = 2n$ (A, CO 1)
12. Derive formula for the divisor sum of Euler totient function. (A, CO 1)
13. Prove that for $n \geq 1$, $\frac{1}{6} n \log n < p_n < 12 \left(n \log n + n \log\left(\frac{12}{e}\right)\right)$ where p_n is the n^{th} prime. (A, CO 2)
14. Prove that for $x \geq 2$, $\pi(x) = \dfrac{\vartheta(x)}{\log x} + \displaystyle\int_2^x \frac{\vartheta(t)}{t \log^2 t} dt$. (A, CO 2)
15. Prove that every Euclidean domain is a unique factorization domain. (An, CO 3)
16. If a domain D is Noetherian, prove that factorization into irreducible is possible in D. (A, CO 3)
17. Let R be a CRU and $\mathfrak{a}$ be an ideal of R. Prove that $\mathfrak{a}$ is prime iff $R/\mathfrak{a}$ is an integral domain. (U, CO 4)
18. If $\mathfrak{p}$ is a maximal ideal of $\mathfrak{O}$, prove that $\mathfrak{p}\mathfrak{p}^{-1} = \mathfrak{O}$. (A, CO 5)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Show that the set of multiplicative functions is a subgroup of the group of all arithmetical functions f with $f(1) \neq 0$. (A, CO 1)

20. Prove that the following statements are equivalent

    1. $\lim_{x \to \infty} \frac{\pi(x) \log x}{x} = 1.$
    2. $\lim_{x \to \infty} \frac{\vartheta(x)}{x} = 1.$
    3. $\lim_{x \to \infty} \frac{\psi(x)}{x} = 1.$

    (A, CO 2)

21. Prove that $\mathfrak{O}$ of $\mathbb{Q}\left(\sqrt{d}\right)$ is not a unique factorization domain where $d = 10, 15, 26, 30$ (A, CO 3)

22. Prove that $\mathrm{N}(\mathfrak{a}\mathfrak{b}) = \mathrm{N}(\mathfrak{a})\mathrm{N}(\mathfrak{b})$, for any ideals $\mathfrak{a}, \mathfrak{b}$ of $\mathfrak{O}$. (An, CO 5)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1		U	1, 2, 3, 11, 12, 19	12
CO 2		A	4, 5, 6, 13, 14, 20	12
CO 3		A	15, 16, 21	9
CO 4		An	7, 8, 9, 17	5
CO 5		An	10, 18, 22	8

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;