Reg. No .... Name **23P2031**

M. Sc. DEGREE END SEMESTER EXAMINATION : MARCH 2023

SEMESTER 2 : MATHEMATICS

COURSE : **21P2MATT08 : GRAPH THEORY**

(For Regular - 2022 Admission and Supplementary - 2021 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Let G be a 2-connected graph and u and v be vertices of G. Let P be a $u-v$ path in G.Is it necessarily true that there exists another $u-v$ path in G such that P and Q are internally disjoint? Justify your answer. (An, CO 1)
2. Give an example of a degree sequence that is realizable as the degree sequence by only a disconnected graph. (Cr, CO 1)
3. Define a graphical sequence.Show that the sequence $(7,6,3,3,2,1,1,1)$ is not graphical. (U, CO 1)
4. Give an example of a tree with disjoint center and centroid. (An, CO 2)
5. Determine κ, λ and δ of K_6. (E, CO 2)
6. Prove or disprove: If closure of G is Hamiltonian, then G is Hamiltonian. (A, CO 2)
7. Define an Eulerian graph.Give examples of Eulerian and non-Eulerian graphs. (U, CO 3)
8. Give an example of a cubic graph with edge chromatic number 4. (A, CO 3)
9. Compute $\chi'(C_9)$. (A, CO 4)
10. Determine $\chi'(K_9)$. (A, CO 4)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Show that the sequence $6,6,5,4,3,3,1$ is not graphical. (An, CO 1)
12. Show that every tournament of order n has at most one vertex with $d^+(v)=n-1$. (A, CO 1)
13. Determine κ, λ and δ of W_7. (E, CO 2)
14. Prove that a vertex v of a tree T with at least three vertices is a cut vertex of T if and only if v is not a pendant vertex. (A, CO 2)
15. Prove that if G is Hamiltonian, prove that for every nonempty proper subset S of $V, \omega(G-S) \leq |S|$. (E, CO 3)
16. Briefly describe the Konigsberg Bridge problem and its significance. (R, CO 3)
17. Show that a graph is 3-critical if and only if it is an odd cycle. (An, CO 4)
18. Show that a simple cubic graph with a cut edge is 4 edge chromatic. (An, CO 4)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Define automorphism of a simple graph G. Show that the set $\Gamma(G)$ of all automorphisms of a simple graph G is a group with respect to the compositions of mappings as the group operation. Further show that for any simple graph G, $\Gamma(G) = \Gamma(G^c)$ (A, CO 1)

20. Show that every tree has a center consisting of either a single vertex or two adjacent vertices. (U, CO 2)

21. Show that a graph is Eulerian if and only if each edge e of G belongs to an odd number of cycles of G. (An, CO 3)

22. Show that every planar graph is 5-vertex colorable. (An, CO 4)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Explain basic concepts such as subgraphs, degrees of vertices, paths and connectedness, automorphisms of a simple graph, line graphs and basic concepts of tournaments.	E	1, 2, 3, 11, 12, 19	12
CO 2	Comprehend connectivity, vertex cuts , edge cuts, connectivity and edge connectivity, blocks, counting the number of spanning trees and Cayley's formula.	E	4, 5, 6, 13, 14, 20	12
CO 3	Analyse vertex and edge independent sets, Eulerian graphs, Hamiltonian graphs, vertex colorings and related results.	E	7, 8, 15, 16, 21	11
CO 4	Explain edge coloring and planarity, certain definitions and properties, dual of a plane graph, the four color theorem and the Heawood five color theorem.	E	9, 10, 17, 18, 22	11

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;