Reg. No .... Name **23P2018**

M. Sc DEGREE END SEMESTER EXAMINATION : MARCH 2023

SEMESTER 2 : MATHEMATICS

COURSE : 21P2MATT07: COMPLEX ANALYSIS

(For Regular - 2022 Admission and Supplementary - 2021 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Show that $\lim_{x\to\infty} n^{1/n} = 1$ (U, CO 1)
2. Prove that $e^{-z} = \frac{1}{e^z}$ (A, CO 1)
3. Prove that $|e^z| = exp(Rez)$ (A, CO 1)
4. Evaluate $\int_\gamma \frac{e^{iz}}{z^2} dz, \gamma(t) = e^{it}, 0 \le t \le 2\pi$ (A, CO 2)
5. State the second and third version of Cauchy's theorem (R, CO 2)
6. Let γ be a rectifiable curve in $\mathbb{C}$ and suppose that F_n and F are continuous functions on γ. If $F = u - \lim F_n$ on γ then prove that $\int_\gamma F = \lim \int_\gamma F_n$ (U, CO 2)
7. State and prove open mapping theorem. (R, CO 3)
8. Define removable singularity? Give an exammple of a function with a removable singularity and non removable singularity. (U, CO 3)
9. Define convex set and convex function with examples in a complex plane (A, CO 4)
10. State Hadamard's three circle theorem (R, CO 4)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. If G is open and connected and $f : G \to C$ is differentiable with $f'(z) = 0$ for all z in G, then f is constant. (U, CO 1)
12. Let G be either the whole plane $\mathbb{C}$ or some open disk. If $u : G \to \mathbb{R}$ is a harmonic function then prove that u has a harmonic conjugate. (A, CO 1)
13. State and prove Abel's theorem (A, CO 2)
14. Show that $\int_0^{2\pi} \frac{e^{is}}{e^{is}-z} dx = 2\pi$ (A, CO 2)
15. Let $f(z) = \frac{1}{z(z-1)(z-2)}$ give the Laurent series exapansion of $f(z)$ in a) $ann(0, 0, 1)$ b) $ann(0, 1, 2)$ c) $ann(0, 2, \infty)$ (A, CO 3)
16. Obtain the Laurent's series expansion of $f(z) = \frac{z^2-1}{(z+2)(z+3)}$ in the region a) $|z| \le 2$ b) $2 \le |z| \le 3$ c) $|z| \ge 3$ (A, CO 3)
17. State and prove the third version of the Maximum modulus theorem (R, CO 4)
18. State and prove the first version of The Maximum Modulus theorem (U, CO 4)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. If $T(z) = \frac{az+b}{cz+d}$ show that $T(\mathbb{R}_\infty) = \mathbb{R}_\infty$ if and only if we can choose a, b, c, d to be real numbers (An, CO 1)

20. State and prove Leibniz's theorem (R, CO 2)

21. Show that $\int_{\infty}^{-\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{(2)}}$ (A, CO 3)

22. Let $a \leq b$ and let G be the vertical strip $\{x + iy/a \leq x \leq b\}$. Suppose $f : \bar{G} \longrightarrow \mathscr{C}$ is continuous and f is anaytic in G. If we define $M; [a, b] \longrightarrow \mathscr{R}$ by $M(x) = sup\{|f(x + iy)|/ - \infty \leq y \leq \infty\}$ and $|f(z)| \leq B$ for all z in G then prove that $logM(x)$ is a convex function. (An, CO 4)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Analyze Analytic functions and Mobius transformations	U	1, 2, 3, 11, 12, 19	12
CO 2	Explain power series representation of analytic functions, Cauchy’s integral formula, Cauchy’s theorem	U	4, 5, 6, 13, 14, 20	12
CO 3	Illustrate about different types of singularities, residues and Rouche’s theorem	U	7, 8, 15, 16, 21	11
CO 4	Explain Maximum Modulus theorem, maximum principle, Schwarz’s lemma, convex functions and Hadmard’s Three Circles Theorem.	U	9, 10, 17, 18, 22	11

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;