Reg. No .... Name **23P2003**

M. Sc. DEGREE END SEMESTER EXAMINATION : MARCH 2023

SEMESTER 2 : MATHEMATICS

COURSE : **21P2MATT06 : BASIC TOPOLOGY**

(For Regular - 2022 Admission and Supplementary - 2021 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1.	Does $int(A \cup B) = int(A) \cup int(B)$ hold for every subset A, B of a space X?	(A, CO 1)
2.	Find the closure of $A = \{0\}$x$[-1, 1]$ as a subset of R^2 with the standard topology?	(R, CO 1)
3.	Define hereditary in a topological space. Is second countability a hereditary property?	(U, CO 1)
4.	Write an example of a divisible property in a topological space.	(R, CO 2)
5.	Is $A = [0, 1]$ compact subset of $\mathscr{R}$, under the usual topology? Justify.	(A, CO 2)
6.	Define a lebesgue number.	(R, CO 2)
7.	Prove that every non empty connected subset is contained in a unique component.	(U, CO 3)
8.	Is the set of rational numbers connected ? Justify.	(A, CO 3)
9.	Define (i) T_2 space (ii) Completely regular Space.	(R, CO 4)
10.	Define normal space. Give an example.	(R, CO 4)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11.	Prove that a sequence in a co-finite space is convergent if and only if there exist 1 term which repeats infinitely many times.	(U, CO 1)
12.	Define metric topology. Prove that every metric space is a topological space, where the topology is metric topology.	(R, CO 1)
13.	Prove that every second countable space is Lindeloff.	(R, CO 2)
14.	Suppose (X, τ) be a space and $Y \in \tau$. Prove that a subset B of Y is open in Y if and only if it is open in X.	(R, CO 2)
15.	A space is locally conneceted at a point x if and only if for every neighbourhood N of x, the component of N containing x is a neighbourhood of x.	(R, CO 3)
16.	Prove that the topological product of two connected space is connected.	(U, CO 3)
17.	Prove that all metric spaces are T_4.	(U, CO 4)
18.	Prove that the real line with semi open interval topology is normal.	(A, CO 4)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. State and prove the closure axiom on a topological space $(X, \mathscr{T})$. (U, CO 1)

20. Suppose $f : X \to Y$ is continuous at $x_0 \in X$, where X and Y are topological spaces. Prove that whenever $\{x_n\}$ converges to $x_0\ inX$, sequence $\{f(x_n)\}$ converges to $f(x_0)\ inY$. (U, CO 2)

21. (a)Establish five equivalent conditions for a space to be connected.(b) Prove that every space is a disjoint union of its components. (U, CO 3)

22. If A, B be compact subsets of topological spaces X, Y respectively and W be an open subset of $X \times Y$ containing the rectangle $A \times B$, then prove that there exists open sets U, V in X, Y respectively such that $A \subset U, B \subset V$ and $U \times V \subset W$. (U, CO 4)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Classify and compare different topological spaces	U	1, 2, 3, 11, 12, 19	12
CO 2	Continuous functions and Quotient map	U	4, 5, 6, 13, 14, 20	12
CO 3	Connectedness	U	7, 8, 15, 16, 21	11
CO 4	Separation Axiom	U	9, 10, 17, 18, 22	11

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;