Reg. No .... Name **22P1046**

M. Sc. DEGREE END SEMESTER EXAMINATION : OCTOBER 2022

SEMESTER 1 : MATHEMATICS

COURSE : **21P1MATT04: ORDINARY DIFFERENTIAL EQUATIONS**

(For Regular - 2022 Admission and Supplementary - 2021 Admission)

Duration : Three Hours Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. State Sturm Seperation theorem. (R, CO 1)
2. Show that the zeroes of the functions $a\sin x + b\cos x$ and $c\sin x + d\cos x$ are distinct and occur alternately whenever $ad - bc \neq 0.$ (A)
3. If a function f(x) has a series of the form $f(x) = \sum_{n=1}^{\infty} a_n J_p(\lambda_n x)$, find the value of a_n. (A)
4. Calculate $\left(\frac{3}{2}\right)!$ (U)
5. Express $2 - 3x + 4x^2$ in terms of Legendre polynomials. (A)
6. Find the critical points of $\frac{dx}{dt} = -x$, $\frac{dy}{dt} = 2x^2y^2.$ (U)
7. Find the critical points $\frac{d^2x}{dt^2} + \frac{dx}{dt} - \left(x^3 + x^2 - 2x\right) = 0.$ (A)
8. Define a positive definite function with an example. (R, CO 3)
9. Show that $f(x, y) = xy$ satisfies a Lipschitz condition on any strip $a \le x \le b, \ -\infty < y < \infty.$ (A)
10. If S is defined by the rectangle $|x| \le a, \ |y| \le b$, show that $f\left(x, y\right) = x^2 + y^2$ satisfies the Lipschitz condition. (U)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Solve $a^2u_{xx} = u_{tt}$ with boundary conditions $u(0, t) = 0, \ u(\pi, \mathrm{t}) = 0$ and initial conditions $u(x, 0) = f(x), \ u_t(x, 0) = g(x).$ (An)
12. Find the eigenvalues and eigenfunctions for the equation y''+λy=0, y(-L)=0, y(L)=0, L>0. (An, CO 1)
13. Show that for n=1,2,3,... , $\left|J_n\left(x\right)\right| \le \frac{1}{\sqrt{2}}.$ (A)
14. Find the first three terms of the Legendre series of $f\left(x\right) = \begin{cases} 0, & -1 \le x < 0 \\ 1, & 0 \le x \le 1 \end{cases}.$ (A)
15. Show that if the roots of the auxillary equation are real, distinct and of the opposite signs for a linear autonomous system , then the critical point is a saddle point. (An, CO 3)

16. Determine the nature and stability properties of the critical point (0, 0) for the linear autonomous system
$\frac{dx}{dt} = 2x,\ \frac{dy}{dt} = 3y.$ (A, CO 3)

17. Apply Picard's method to find the first two approximations to the initial value problem
$y'(x) = x + z,\ z'(x) = x - y^2,\ y(0) = 2,\ z(0) = 1.$ (A)

18. Apply Picard's method to find the first three approximations to the initial value problem $y' = x + y,\ y(0) = 0.$ (A)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. Derive the general Sturm Liouville problem. (Cr, CO 1)

20. State and prove orthogonality property of Bessel's function. (An)

21. For the following linear system , find the general solution , differential equation of the paths and its solution. Sketch a few paths showing the direction of increasing t and discuss the stability of the critical point (0,0). (A, CO 3)
dx/dt = ax-y
dy/dt = x+ay

22. Let f(x,y) be a continuous function that satisfies a Lipschitz condition on a strip defined by $a \le x \le b,\ -\infty < y < \infty$. If (x_0, y_0) is any point of the strip , then prove that the initial value problem $y' = f(x, y),\ y(x_0) = y_0$ has one and only one solution y=y(x) on the interval $a \le x \le b$. (An)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Summarize the concepts of Sturm Separation theorem and Sturm Liouville problems	A	1, 12, 19	8
CO 3	Analyze the concept of linear and nonlinear systems and their stability	An	8, 15, 16, 21	10

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;