Reg. No .... Name **22P1033**

M. Sc. DEGREE END SEMESTER EXAMINATION : OCTOBER 2022

SEMESTER 1 : MATHEMATICS

COURSE : **21P1MATT03 : REAL ANALYSIS**

(For Regular - 2022 Admission and Supplementary - 2021 Admission)

Duration : Three Hours | Max. Weights: 30

PART A

Answer any 8 questions **Weight: 1**

1. Prove or disprove: there exists a function of bounded variation on $[a,b]$ whose derivative is not bounded on (a,b). (An, CO 1)
2. Define total variation. Is the toatal variation can be 0? (R, CO 1)
3. If $\{f_n\}$and $\{g_n\}$ converges uniformly on a E, prove that $\{f_n+g_n\}$converges uniformly on E. (A, CO 3)
4. Prove that $\lim\limits_{n\to\infty}(1+\frac{1}{n})^n=e$. (A, CO 4)
5. Suppose $f\geq 0$, f is continuous on [a,b] and $\int_a^b f(x)dx=0$. Prove that $f(x)=0,\ \forall x\in[a,b]$. (R, CO 2)
6. If $E(z)=\sum\limits_{n=0}^{\infty}\frac{z^n}{n!}$, prove that $E(0)=1\ \forall x\in\mathbb{R}$ (A, CO 2)
7. If $f\in\mathscr{R}(\alpha)$, prove that $|f|\in\mathscr{R}(\alpha)$. Is converse true? Justify. (A)
8. Prove that every uniformly convergent sequence of bounded functions is uniformly bounded (An, CO 1, CO 2)
9. If f is continous on [a,b], then prove that $f\in\mathscr{R}(\alpha)$ on [a,b] (R)
10. Prove that the set of discontinuities of a monotone function is countable. (An, CO 1)

(1 x 8 = 8)

PART B

Answer any 6 questions **Weights: 2**

11. Suppose $c_n\geq 0$ for 1, 2, 3, ... , $\sum\limits_{1}^{\infty}c_n$ converges, $\{s_n\}$ is a sequence of distinct points in (a, b), and $\alpha(x)=\sum_{n=1}^{\infty}\ c_nI(x-s_n)$. Let f be continuous on [a, b]. Then prove that $\int_a^b fd\alpha=\sum_{n=1}^{\infty}\ c_nf\Big(s_n\Big)$ (A)
12. State and prove Cauchy criterion for uniform convergence of a series of functions. (R, CO 3)
13. Let $\{a_{ij}\}, i=1,2,3,\ldots; j=1,2,3,\ldots$ be a double sequence. If $\sum\limits_{j=1}^{\infty}|a_{ij}|=b_i, i=1,2,3,\ldots$ and $\sum b_i$ converges, prove that
$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}a_{ij}=\sum_{j=1}^{\infty}\sum_{i=1}^{\infty}a_{ij}$$
(U, CO 4)
14. Suppose K is compact, and
(a)$\{f_n\}$ is a sequence of continuous functions on K, (U)

(b) $\{f_n\}$converges pointwise to a continuous function f on K,
(c)$f_n(x) \geq f_{n+1}(x)$ for all $x \in K,\ n = 1, 2, 3 \ldots$Then prove that
$f_n \to f$ uniformly on K.

15. State and prove the theorem for change of variable in integration. (A, CO 2)

16. Define equivalent paths. State and prove a necessary and sufficient condition for equivalence of two paths which are one to one on its domain. Give an example for non-equivalent paths (Cr, CO 1)

17. Prove that $f \in \mathscr{R}(\alpha)$ on [a,b] iff for every $\epsilon > 0$, there exists a partition P such that
$U(P, f, \alpha) - L(P, f, \alpha) < \epsilon.$ (U, CO 2)

18. State and prove the additive property of total variation of a function of bounded variation. (An, CO 1)

(2 x 6 = 12)

PART C

Answer any 2 questions **Weights: 5**

19. a) Suppose α increases on [a,b], $a \leq x_0 \leq b$, α is continuous at x_0, $f(x_0) = 1$, and $f(x) = 0$ if $x \neq 0$. Prove that $f \in \mathscr{R}(\alpha)$ and that $\int f d\alpha = 0$
b) Suppose $f \geq 0$, f is continuous on [a,b], and $\int_a^b f(x)dx = 0$. Prove that $f(x) = 0$ for all $x \in [a, b]$. (A, CO 2)

20. a) Define the total variation of a function of bounded variation. Prove that the total variation is zero iff f is constant.
b) State and prove the additive property of total variation of a function of bounded variation. (An, CO 1)

21. Suppose the series $\sum_{n=0}^{\infty} a_n x^n$ and $\sum_{n=0}^{\infty} b_n x^n$ converge in the segment $S = (-R, R)$. Let E be the set of all $x \in S$ at which $\sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} b_n x^n$. If E has a limit point in S, then prove that $a_n = b_n$ for $n = 0, 1, 2, \ldots$. (R, CO 4)

22. Prove that the series $\sum \frac{x^2 + n}{n^2}$ converges uniformly in every bounded interval, but does not converge absolutely for any value of x. (U, CO 3)

(5 x 2 = 10)

OBE: Questions to Course Outcome Mapping

CO	Course Outcome Description	CL	Questions	Total Wt.
CO 1	Explain the functions of bounded variations, rectifiable curves, paths and equivalence of paths.	U	1, 2, 8, 10, 16, 18, 20	13
CO 2	Illustrate the properties of Riemann-Stieljes integral.	An	5, 6, 8, 15, 17, 19	12
CO 3	Analyze the uniform convergence of a sequence of functions with continuity, integrability, differentiability.	An	3, 12, 22	8
CO 4	Apply the properties of power series to exponential, logarithmic and trigonometric functions.	A	4, 13, 21	8

Cognitive Level (CL): Cr - CREATE; E - EVALUATE; An - ANALYZE; A - APPLY; U - UNDERSTAND; R - REMEMBER;