Reg. No .... Name **23U621**

B. Sc. DEGREE END SEMESTER EXAMINATION : MARCH 2023

SEMESTER 6 : MATHEMATICS

COURSE : **19U6CRMAT10 : COMPLEX ANALYSIS**

(For Regular - 2020 Admission and Supplementary - 2019 Admission)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Find the real and imaginary parts of the function $f(z) = z^2$.
2. Define Jordan curve.
3. State the principle of deformation of paths.
4. Define Cauchy principal value of an improper integral.
5. If a series of complex numbers converges, show that the n^{th} term converges to zero as n tends to infinity.
6. Use definition to evaluate $f'(0)$, where $f(z) = \bar{z}$.
7. Evaluate $\lim_{n\to\infty}\left(-2 + i\frac{(-1)^n}{n^2}\right)$.
8. Explain the convergence of an improper integral.
9. Define residue of a function at a point.
10. Write the defintion of the derivative of a function $f(z)$ at $z = z_0$.
11. Compute the Maclaurin series expansion of $f(z) = \sinh z$.
12. State Cauchy-Goursat theorem.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Find the harmonic conjugate of $u(x, y) = x^2 - y^2$.
14. Compute the Maclaurin series expansion of $f(z) = z^2e^{3z}$.
15. Show that if $|f(z)|$ is a constant throughout the domain D, then $f(z)$ must be constant throughout D.
16. Classify the singularity of $f(z) = \dfrac{z^2 - 2z + 3}{z - 2}$
17. State and prove Liouville's theorem.
18. Use Cauchy integral formula to evaluate $\int_C \dfrac{dz}{z - z_0}$, where z_0 be any point interior to a positively oriented simple closed contour C.
19. True or false: Let $z_n = r_n e^{i\theta_n}$ and $z = re^{i\theta}$. Then $z_n \to z$ if and only if $r_n \to r$ and $\theta_n \to \theta$ as $n \to \infty$. Justify.
20. Find the residues of the singularities of $f(z) = \dfrac{z + 1}{z^2 + 9}$.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. State and prove Taylor's theorem.
22. Derive Cauchy-Riemann equations.
23. State and prove Cauchy's residue theorem.
24. Let C denote positively oriented simple closed contours. If a function f is analytic, then prove that $f^n(z) = \frac{n!}{2\pi i} \int_C \frac{f(s)ds}{(s-z)^{n+1}}$.

(10 x 3 = 30)