Reg. No .... Name **23U655**

B. Sc. DEGREE END SEMESTER EXAMINATION : MARCH 2023

SEMESTER 6 : MATHEMATICS

COURSE : **19U6CRMAT12: FOURIER SERIES, LAPLACE TRANSFORMS AND METRIC SPACES**

(For Regular - 2020 Admission and Supplementary - 2019 Admission)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Explain periodic functions. Sketch the graph of the periodic function $f(x) = x$ with period 2π from -∞ to ∞
2. Write the fourmula for half range Fourier cosine series of a function defined in the interval $[0, l]$
3. Find the Fourier coefficient a_0 for the function $f\left(x\right) = \frac{1}{4}(\pi - x)^2,\ 0 < x < 2\pi$
4. Find the inverse Laplace transform of $\frac{s+1}{s^2+s+1}$.
5. Show that $L(sinktsinhkt) = \frac{2k^2s}{s^4+4k^4}$.
6. Define Laplace transform and find the Laplace transform of $\sin x$
7. Give an example of a set (a) which contains a point which is not a limit point of the set, (b) which contains no point which is not a limit point.
8. Prove that the empty set and the full set in a metric space are closed.
9. Define distance of a point from a set and diameter of a set in terms of metric.
10. When do we say that a function is uniformly continuous? Give an example.
11. Differentiate limit and limit point.
12. Define continuity of a function from a metric space to another metic space.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Find the Fourier series to represent e^{ax} in the interval $-\pi < \mathrm{x} < \pi$
14. Find the Fourier sine and cosine series of $f(x) = x + x^2, 0 \leq x \leq 1$.
15. Find the inverse Laplace transform of $\left\{\tan^{-1}\left(\frac{2}{s}\right)\right\}$
16. Find the inverse Laplace transform of $\frac{2s^2-1}{(s^2+1)(s^2+4)}$.
17. Prove that each open sphere is an open set in a metric space.
18. Let X be a metric space and A be a subset of X. Prove that
    1) A is open $\Leftrightarrow$ A=Int(A)
    2) Int(A) is an open subset of A which contains every open subset of A.
19. Let X and Y be metric spaces and fa mapping of X into Y. Prove that if f is continuous, then $f^{-1}(G)$ is open in X whenever G is open in Y.
20. Prove that if a complete metric space is the union of a sequence of its subsets, then the closure of atleast one set in the sequence must have non-empty interior.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Find the Fourier series to represent the function $f(x)$ given by $f\left(x\right) = \begin{cases} x & 0 \le x \le \pi \\ 2\pi - \text{x} & \pi \le \text{x} \le 2\pi \end{cases}$. Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$

22. a) Solve $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t,\ x\left(0\right) = 0, x'\left(0\right) = 1$

    b) Apply convolution theorem to find the inverse Laplace transform of $\frac{s^2}{s^4 - a^4}$

23. Prove that arbitrary union of open sets is open and that finite intersection of open sets is open.

24. (a) Define completeness of a metric space.

    (b) Prove that a subspace of a complete metric space is complete $\Leftrightarrow$ it is closed.

(10 x 3 = 30)