Reg. No .... Name **22U630**

B. Sc. DEGREE END SEMESTER EXAMINATION - MARCH 2022

SEMESTER 6 : MATHEMATICS

COURSE : **19U6CRMAT11: LINEAR ALGEBRA AND GRAPH THEORY**

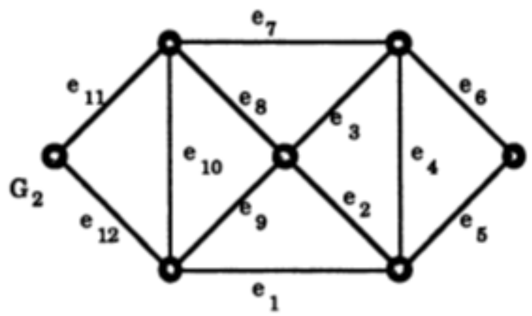
(For Regular - 2019 Admission)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Determine whether the set of all 2×2 real matrices is a vector space under regular scalar multiplication but with vector addition defined to be matrix multiplication.That is, $u \odot v = uv$
2. Prove that in any vector space V, $\alpha \odot \mathbf{0} = \mathbf{0}$, for every scalar α .
3. Prove that for any vector w in a vector space V, $\mathbf{1 \odot w = - w}$.
4. Define functions.
5. Define the following terms;
 a) Domain
 b) Range
 c) Image
6. Find f (2), f (5), and f (-5) for $f(x) = 1/x^2$
7. define a graph
8. Draw the graph whose adjacency matrix is
$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$
9. State and prove first theorem on graph theorey
10. Check wheather the following graph is Euler



11. Draw K_5 and mark a maximum matching in the graph
12. Define a maximal non Hamiltonian graph with an example

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

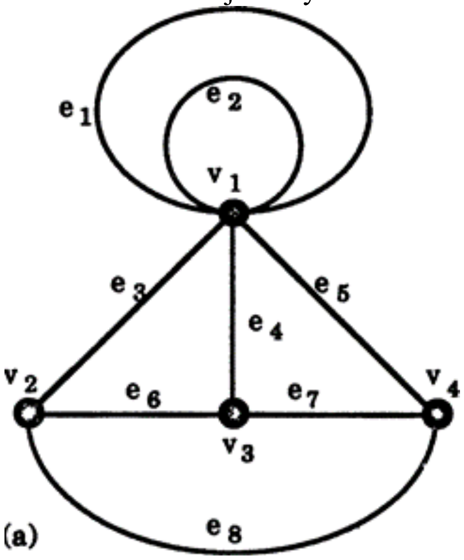
13. Determine whether $\mathbb{S} = \{p(t) \in \mathbb{P}^2 \mid p(2) = 0\}$ is a subspace of P^2.
14. Determine whether the set of two-dimensional column matrices with all components real and equal is a vector space under regular addition but with scalar multiplication defined as

$$\alpha \odot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -\alpha a \\ -\alpha b \end{bmatrix}$$

15. Determine whether the transformation **L** is linear if **L: $R^2 \to R^1$** is defined by L [a b] = ab for all real numbers a and b.

16. Determine whether the transformation **L** is linear if **L: $R^2 \to R^2$** is defined by L [a b] = [a+2 b-2] for all real number s a and b.

17. Write down the adjacency matrix and the incidence matrix for the graph



18. Prove that a graph G is connected if and only if G has a spanning tree

19. Prove that a simple graph G is Hamiltonian if and only if its closure $c(G)$ is Hamiltonian

20. Explain Konigsberg bridge problem and its solution

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Find a basis for the span of the vectors in

$$\mathbb{C} = \{t^3 + 3t^2,\ 2t^3 + 2t - 2,\ t^3 - 6t^2 + 3t - 3,\ 3t^2 - t + 1\}$$

22. Find matrix representations for the linear transformation T : $R^2 \to R^2$ is defined by

$$T\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 11a + 3b \\ -5a - 5b \end{bmatrix}$$

(a) with respect to the standard basis C = { $[1 \ \ 0]^T$, $[0 \ \ 1]^T$ }

(b) with respect to the basis D = { $[1 \ \ 1]^T$, $[1 \ \ -1]^T$ }

(c) with respect to the basis E = { $[3 \ \ -1]^T$, $[1 \ \ -5]^T$ }

23. State and prove Whitney's theorem

24. State and prove Dirac theorem

(10 x 3 = 30)