Reg. No………………………………………….  Name……………………………………………  **21U655**

# B. Sc. DEGREE END SEMESTER EXAMINATIONS – APRIL 2021

## SEMESTER – 6: MATHEMATICS (CORE COURSE)

## COURSE: 15U6CRMAT13: OPERATIONS RESEARCH

*(Common for Regular 2018 admission & Improvement 2017/Supplementary 2017/2016 /2015 admissions)*

Time: Three Hours  Max Marks: 75

### SECTION A

***Answer all questions***

1. Define a basic feasible solution to an LPP.
2. Find the convex hull of the points (1,0,0), (0,1,0), (0,0,1) in $E_3$.
3. Define linear independence of vectors.
4. Write the dual of the following LPP: Minimize $Z = 6x_1 + 3x_2 + 9x_3$
   Subject to $3x_1 + 4x_2 + x_3 \geq 5$
   $6x_1 - 3x_2 + x_3 \leq 10$
   $x_1, x_2 \geq 0$ , $x_3$ is unrestricted.
5. State the complementary slackness condition.
6. Define a loop in a transportation problem.
7. Give the mathematical formulation of an assignment problem.
8. Define transient state and steady state.
9. Define a waiting line.
10. What is traffic intensity. (1 x 10 = 10)

### SECTION B

***Answer any Eight questions***

11. Express the following problem in the standard form:
    Maximize $Z = 3x_1 - 2x_2 + 4x_3$ subject to
    $x_1 + 2x_2 + x_3 \leq 8$ , $2x_1 - x_2 + x_3 \geq 2$ , $4x_1 - 2x_2 - 3x_3 = -6$ , $x_1, x_2, x_3 \geq 0.$
12. Define slack and surplus variables.
13. Give an example of convex set with (i) no vertex (ii) one vertex only.
14. State the general rules for converting any primal LPP into its dual.
15. Explain degeneracy in LPP.
16. State the difference between transportation problem and assignment problem.
17. State the necessary condition for the existence of feasible solution to the transportation problem.
18. Write the relation between (i) $W_s$ and $W_q$ (ii) $L_s$ and $L_q$ .
19. Explain queue discipline and its various forms.
20. Arrival rate of telephone calls at a telephone booth are according to Poisson distribution, with an average time of 9 minutes between two consecutive arrivals. The length of telephone call is assumed to be exponentially distributed, with mean 3 minutes. What is the probability that an arrival will have to wait for more than 10 minutes before the phone is free. (2 x 8 =16)

## SECTION C

***Answer any Five questions***

21. Prove that the optimum value of $f(X)$ of the primal, if it exists is equal to the optimum value of $\emptyset(Y)$ of the dual.
22. Solve graphically: Maximize$Z = 2x_1 + 3x_2$ subject to

$$x_1 - x_2 \leq 2, x_1 + x_2 \geq 4, x_1, x_2 \geq 0.$$

23. Prove that the set of feasible solution $S_F$ , if not empty is a closed convex set bounded from below and so has atleast one vertex.
24. Obtain an initial basic feasible solution to the following transportation problem:

| | D | E | F | G | |
|---|---|---|---|---|---|
| A | 11 | 13 | 17 | 14 | 250 |
| B | 16 | 18 | 14 | 10 | 300 |
| C | 21 | 24 | 13 | 10 | 400 |
| | 200 | 225 | 275 | 250 | |

25. Prove that the transportation problem has a triangular basis.
26. Explain the characteristics of the arrivals of queuing system.
27. A super market has a single cashier. During the peak hours, customers arrive at a rate of 20 customers per hour. The average number of customers that can be processed by the cashier is 24 per hour. Calculate (i) the probability that the cashier is idle.
    (ii) the average number of customers in the queuing system.
    (iii) the average time a customer spends in the queue waiting for service.

(5 x 5 = 25)

## SECTION D

***Answer any Two questions***

28. Solve Minimize $Z = 2x_1 + x_2$ subject to

$$3x_1 + x_2 = 3,\ 4x_1 + 3x_2 \geq 6,\ x_1 + 2x_2 \leq 3\ , x_1, x_2 \geq 0.$$

29. Solve the following problem by the dual simplex method.
    Maximize $Z = -3x_1 - 2x_2$ subject to

$$x_1 + x_2 \geq 1,\ x_1 + x_2 \leq 7, x_1 + 2x_2 \geq 10\ ,\ x_2 \geq 3\ \ , x_1, x_2 \geq 0.$$

30. Four operators, A, B, C, D are to be assigned to four machines $M_1, M_2, M_3, M_4$ with the restriction that A and C cannot work on $M_3$ and $M_2$ respectively. The assignment costs are given below. Find the minimum assignment cost.

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ |
|---|---|---|---|---|
| A | 5 | 2 | - | 5 |
| B | 7 | 3 | 2 | 4 |
| C | 9 | - | 5 | 3 |
| D | 7 | 7 | 6 | 2 |

31. Explain the probability distributions in queuing system. (12 x 2 = 24)