Reg. No .... Name **20U349**

B. Sc. DEGREE END SEMESTER EXAMINATION - OCT. 2020 : JANUARY 2021

SEMESTER 3 : COMPUTER APPLICATION

COURSE : **19U3CRCMT4 : VECTOR CALCULUS, TRIGONOMETRY AND MATRICES**

(For Regular - 2019 Admission)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Find gradϕ when ϕ is given by $\phi=3x^2y-y^3z^2$ at the point (1,-2,-1)?
2. If A and B are vector functions then $\nabla\times(A+B) = \nabla\times A + \nabla\times B$
3. Show that the vector field V=(sin y + z)**i**+(x cos y-z)**j**+(x - y)**k** is irrotational.
4. If **A**(t) =$(3t^2-2t)$**i**+(6t-4)**j**+4t**k**, evaluate $\int_2^3 A\left(t\right)dt$.
5. If **r**=t**i**-t^2**j**+(t-1)**k** and **S**=$2t^2$**i**+6t**k**, evaluate $\int_0^2 (\mathrm{r}.\mathrm{S})\,\mathrm{d}\,\mathrm{t}$
6. State Stoke's theorem.
7. Separate into real and imaginary parts the expression tan(x+iy).
8. Prove that $\sin h\ 2x = \frac{2\ \tan h\ x}{1-\tan h^2\ x}$
9. If x is real, show that $\cos h^{-1}\ x\ = \log\ \left[x\ + \sqrt{x^2-1}\right]$.
10. Show how that the vectors x_1=(1,2,4), x_2=(2,-1,3), x_3=(0,1,2), and x_4=(-3,7,2) are linearly dependent and find the relation between them.
11. Show that every square matrix is expressible as the sum of a Hermitian matrix and a skew-Hermitian matrix.
12. Find the eigen values of the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Show that the vector field defined by **F**=$2xyz^3$**i**+x^2z^3**j**+$3x^2yz^2$**k** is irrotational. Find the scalar potential u such that **F**=grad u**.**
14. Find the directional derivative of the function $f(x,y,z)=xy^2+yz^3$ at the point (2,-1,1) in the direction of the vector **i**+2**j**+2**k.**
15. If **r**(t)=$5t^2$**i**+t**j**-t^3**k**, prove that $\int_1^2 \left(\mathrm{r}\times\frac{\mathrm{d}^2\mathrm{r}}{\mathrm{dt}^2}\right)\mathrm{d}\,\mathrm{t}$=-14**i**+75**j**-15**k**.
16. Evaluate $\iint_s F.\hat{n}ds$ where F= $4xi-2y^2j+z^2k$ and S is the surface bounding the region $x^2+y^2=4,z=0,z=3$.
17. If sin(A+iB)=x+iy, show that a) $\frac{x^2}{\cos h^2 B}+\frac{y^2}{\sin h^2 B}=1.$.

    b) $\frac{x^2}{\sin^2 A}-\frac{y^2}{\cos^2 A}=1.$
18. Separate into real and imaginary parts the expression $\sin^{-1}(\cos\theta+i\sin\theta)$, where θ is real.

19. For what values of a and b do the system of equations:

$x + 2y + 3z = 6, \quad x + 3y + 5z = 9, \quad 2x + 5y + az = b$

have (i) no solution (ii) unique solution (iii) more than one solution?

20. Find the inverse of A = $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$ by Guass Jordan method.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. If **r** =xi+yj+zk, prove that

i)div(r^n **r**) = (n+3) r^n

ii)curl(r^n **r**) = **0**

iii)$\nabla^2\left(\frac{1}{r}\right) = 0$

iv)$\frac{\vec{r}}{r^3}$ is solenoidal.

22. Verify divergence theorem for **F**=$2x^2y$**i**-y^2**j**+$4xz^2$**k** taken over the region in the first octant bounded by $y^2+z^2=9$ and x=2.

23. Find the sum to infinity of the following series.

a) sinα.cosα +$\sin^2α$.cos2α+$\sin^3α$.cos3α+............∞.

b)$\sin\ \alpha - \frac{\sin\ (\alpha+2\beta)}{2!} + \frac{\sin\ (\alpha+4\beta)}{4!} - \ldots\ldots\ldots\ldots\ldots\ldots\ldots\infty$

24. Find the characteristic equation of the matrix A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$. Show that the equation is satisfied by A and hence find A^{-1}.

(10 x 3 = 30)