

B. Sc. DEGREE END SEMESTER EXAMINATION - OCT. 2020 : JANUARY 2021
SEMESTER 3 : COMPLEMENTARY MATHEMATICS FOR B Sc PHYSICS/CHEMISTRY
COURSE : 19U3CPMAT3 : DIFFERENTIAL EQUATIONS, MATRICES AND TRIGONOMETRY
(For Regular - 2019 Admission)

Time : Three Hours

Max. Marks: 75

PART A**Answer any 10 (2 marks each)**

1. Verify whether $ydx - xdy + 3x^2y^2e^{x/3}dx = 0$ is exact.
2. Find the differential equations of all circles touching the axis of y at the origin and centres on the axis of x .
3. Solve $(x + y)dx - xdy = 0$.
4. Show that $\sin 6x = 6 \cos^5 x \sin x - 20 \cos^3 x \sin^3 x + 6 \cos x \sin^5 x$.
5. Prove that $\sin^5 x = \frac{1}{16} [\sin 5x - 5 \sin 3x + 10 \sin x]$.
6. Prove that $\cosh 2x = 1 + 2 \sinh^2 x$.
7. Find one of the solution of $x(y - z)p + y(z - x)q = z(x - y)$.
8. Identify the dependent and independent variables of the first order PDE $\frac{\partial u}{\partial x} = x^2 + y^2$.
9. Show that $u = e^{x+y}$ is a solution of the PDE $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$.
10. Define Eigen value and eigen vector.
11. Explain elementary transformation.
12. Evaluate the eigen values of $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

(2 x 10 = 20)**PART B****Answer any 5 (5 marks each)**

13. Find the solution of Bernoulli differential equation $\frac{dy}{dx} - y = e^x y^2$.
14. Solve the differential equation of first order $(2y^2 + 3x)dx + 2xydy = 0$
15. Show that $\cos 8\theta = \cos^8 \theta - 28 \cos^6 \theta \sin^2 \theta + 70 \cos^4 \theta \sin^4 \theta - 28 \cos^2 \theta \sin^6 \theta + \sin^8 \theta$.
16. Separate into its real and imaginary parts the quantity $\tan^{-1}(\cos \theta + i \sin \theta)$.
17. Determine the particular solution of the PDE $\log \left(\frac{\partial^2 z}{\partial x \partial y} \right) = x + y$.
18. Form a partial differential equation by eliminating the function f from the relation $z = y^2 + 2f \left(\frac{1}{x} + \log y \right)$.
19. Test for consistency and if consistent solve $x + 2y - z = 3, 3x - y + 2z = 1, 2x - 2y + 3z = 2, x - y + z = -1$
20. Reduce to normal form to evaluate the rank of the the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Form the differential equation of all circles of radius a .
22. Sum the series $c \sin \alpha + \frac{c^2}{2} \sin 2\alpha + \frac{c^3}{3} \sin 3\alpha + \dots$ and $c \cos \alpha + \frac{c^2}{2} \cos 2\alpha + \frac{c^3}{3} \cos 3\alpha + \dots$, where c is numerically not greater than unity.
23. Find the integral curve of (a) $\frac{dx}{x^2(y^3-z^3)} = \frac{dy}{y^2(z^3-x^3)} = \frac{dz}{z^2(x^3-y^3)}$
(b) $xp + yq = 3z$

24. Calculate the eigen values and eigen vectors of the following matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(10 x 3 = 30)