Reg. No……………………………………..  Name……………………………………………

# B. Sc. DEGREE END SEMESTER EXAMINATION - APRIL 2021

SEMESTER **–6: MATHEMATICS (CORE COURSE)**

COURSE: **15U6CRMAT11: DISCRETE MATHEMATICS**

*(Common for Regular 2018 admission & Improvement 2017/Supplementary 2017/2016 /2015 admissions)*

Time: Three Hours  Max Marks: 75

## PART A

***Answer all questions. Each question carries 1 mark.***

1. If the Complete graph Kn has 15 edges find n.
2. Define underlying simple graph of a graph G.
3. State Cayley's theorem.
4. Draw a Hamiltonian graph which is not Euler.
5. State Dirac's Theorem.
6. Draw a graph and mark a maximum Matching in it.
7. Distinguish between enciphering and deciphering.
8. Give an example of a super increasing sequence.
9. Give an example of a Chain.
10. State Duality principle. (1 x 10 = 10)

## PART B

***Answer any eight questions. Each question carries 2 mark.***

11. If G is a k regular graph where k is an odd number then prove that the number of edges in G is a multiple of k.
12. Write the incidence matrix of $K_{1,1}$.
13. Prove that any tree with atleast two vertices is a bipartite graph.
14. Define maximal non Hamiltonian graph. Give an example.
15. Define closure of a simple graph G.
16. Distinguish between perfect matching and maximum matching in a graph.
17. Distinguish between monoalphabetic cipher and polyalphabetic cipher.
18. Solve the super increasing Knapsack Problem $51 = 3x_1 + 5x_2 + 9x_3 + 18x_4 + 37x_5$.
19. Distinguish between greatest element and maximal element of a poset.
20. Show by an example that Union of two sublattices may not be a sublattice. (2 x 8 = 16)

## PART C

***Answer any five questions. Each question carries 5 mark***

21. Prove that a tree with n vertices has precisely n-1 edges.
22. If G is an acyclic graph with n vertices and k connected components, then prove that G has n – k edges.
23. If for each pair of distinct vertices u and v of a simple graph g, there are two internally disjoint u-v paths in G, then prove that G is 2- connected.

24. If G is a graph in which degree of every vertex is atleast two, then prove that G contains a cycle.
25. Prove that if a matching M in a graph G is maximum, then G has no M augmenting Path.
26. Decipher the message 'BBOT XWBZ AWUVGK' which was produced by the autokey cipher with seed RX.
27. Prove that a sublattice S of a lattice L is a convex sublattice if and only if $\forall$ a,b $\epsilon$ S (a $\leq$ b), [a,b] $\subseteq$ S.

(5 x 5 = 25)

**PART D**

***Answer any two questions. Each question carries 12 mark.***

28. If e is an edge of a graph G and if G-e is the subgraph obtained by deleting e from G then Prove that $\omega$ (G) ≤ $\omega$ (G-e) ≤ $\omega$ (G) +1.
29. Prove that a connected graph G is Euler if and only if the degree of every vertex is even.
30. A user of the Knapsack Cryptosystem has a private key consisting of the super increasing sequence 2, 3, 7, 13, 27 the modulus m = 60 and multiplier a = 7.
    (a) Find the users listed public key.
    (b) With the aid of the public key encrypt the message 'SEND MONEY'
31. (a) Prove that product of two lattices is a lattice.
    (c) Prove that a finite lattice has the least and greatest element.

(12 x 2 =24)

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