Reg. No ..... Name **20U324**

B. Sc DEGREE END SEMESTER EXAMINATION - OCT. 2020 : JANUARY 2021

SEMESTER 3 : MATHEMATICS FOR B Sc COMPUTER APPLICATIONS

COURSE : **19U3CRCMT3 : CALCULUS**

(For Regular - 2019 Admission)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Find the radius of curvature of the curve $y = 3x^2 + 4x \; at \; the \; point \; (1, 7)$.
2. $Find \; the \; n^{th} \; derivative \; of \; \cos^3 \; x.$
3. Find the points of inflection of the curve $y = 3x^4 - 4x^3 + 1$.
4. Find $f_x, \; f_y \; and \; f_z \; if \; f\left(x, \; y, \; z\right) = e^{-\left(x^2+y^2+z^2\right)}$.
5. Use chain rule to find $\frac{\partial w}{\partial t} \; at \; t = 0$, $where \; w = \; x^2 + y^2 \; along \; the \; path \; x = \cos \; t + \sin \; t, \; y = \cos \; t - \sin \; t$.
6. $If \; w = x^2 + y - z + \sin \; t \; \; and \; x + y = t, \; \; find \; \left(\frac{\partial w}{\partial x}\right)_{y,z}$.
7. The region between the curve y=2$\sqrt{x}$, $0 \leq x \leq 2$ and the x-axis is revolved about the x-axis to generate a solid. Find its volume using Disk method.
8. The solid lies between the planes perpendicular to the y-axis at y=0 and y=2.The cross-sections perpendicular to the y-axis are circular disks with diameters running from the y-axis to the parabola $x = \sqrt{5}y^2$.Find the volume of the solid.
9. Evaluate the integral $\int_0^{\pi/2} \frac{3 \; \sin x \; \cos x}{\sqrt{1+3 \; \sin^2 \; x}} dx$.
10. $If \; f\left(x, y\right) = x^2y - 2xy \; and \; \; R: \; 0 \leq x \leq 3, \; \; -2 \leq y \leq 0, \; then \; evaluate \; \iint_R f\left(x, y\right) dA.$
11. Evaluate $\iint_R \; \frac{\sin \; x}{x} dA$,where R is the triangle in the xy- plane bounded by the x-axis, the line y = x and the line x = 1.
12. Evaluate $\int_0^1 \; \int_1^2 \left(x^2 + y^2\right) \; dx \; dy.$

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. $Find \; the \; n^{th} \; derivative \; of \; e^{ax} \; \cos^2 \; x \; sin \; x.$
14. Find the points of inflection and the inflectional tangents to the curve $y = \frac{x^3-x}{3x^2+1}$.
15. Find all local maxima, local minima and saddle points of the function $f\left(x, y\right) = x^2 + xy + y^2 + 3x - 3y + 4.$
16. Find positive numbers x,y,z such that x+y+z= 20 and xyz^2 is a maximum.
17. Using disk method, find the volume of the solid generated by revolving the region bounded by the curve $y = x^2$ and the lines y = 0, x = 2 about the x-axis.
18. The region enclosed by the x-axis and the parabola $y = 3x - x^2$ is revolved about the vertical line x=-1 to generate the solid.Find the volume of the solid using shell method.
19. a) Solve the system u = x + 2y and v = x - y for x and y in terms of u and v. Then find the Jacobian J(u, v).
    b) Find the image under transformation u = x+2y and v = x-y of the triangular region in the xy-plane bounded by the lines y = 0, y = x and x+2y = 2. Sketch the transformed region in the uv-plane.
20. Find the area enclosed by the cardioid r = a (1+ cos θ).

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. $If\ \ y = \sin\left(m\ \sin^{-1} x\right),\ show\ that$

$$y_n\left(0\right) = \begin{cases} 0, & if\ n\ is\ even \\ m\left(1-m^2\right)\left(3^2-m^2\right).....\left[(n-2)^2-m^2\right], & if\ n\ is\ odd \end{cases}$$

22. Using Legrange multipliers, find the maximum and minimum values of $f(x,y,z) = x - 2y + 5z$ on the sphere $x^2 + y^2 + z^2 = 30$.

23. a) Find the area of the surface generated by revolving the curve $y = \frac{x^3}{9},\ 0 \le x \le 2,\ about\ the\ x-axis.$

b) Find the volume of the solid generated by revolving the region bounded by the parabola $y = x^2$ and the line y = 1 about the line y =2.

24. Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.

(10 x 3 = 30)