Reg. No .... Name **20U323**

B. Sc DEGREE END SEMESTER EXAMINATION - OCT. 2020 : JANUARY 2021

SEMESTER 3 : MATHEMATICS

COURSE : **19U3CRMAT3 : VECTOR CALCULUS, THEORY OF EQUATIONS AND MATRICES**

(For Regular - 2019 Admission)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Show that $\nabla\left(\overline{A}\times\overline{B}\right)=\left(\nabla.\overline{B}\right)\overline{A}-\left(\nabla.\overline{A}\right)\overline{B}+\left(\overline{B}.\nabla\right)\overline{A}-\left(\overline{A}.\nabla\right)\overline{B}.$
2. Give the physical interpretation of divergence.
3. If $\bar{r}\ xi+yj+zk$,show that$i\Big)grad\ r=\frac{\bar{r}}{r}$

   $$ii\Big)grad\left(\frac{1}{r}\right)=\frac{\overline{-r}}{r^3}$$

   $$iii\Big)\nabla r^n=nr^{n-2}\overline{r.}$$
4. Use divergence theorem to show that $\oint\limits_s \nabla r^2 d\bar{s}=6v$ where s is any closed surface enclosing a volume V.
5. State Stoke's theorem.
6. State Gauss divergence theorem and prove that for any closed surface $s,\iint_s curl\ \overline{F}.\widehat{n}ds=0.$
7. Show that any real polynomial equation of odd degree has at least one real root.
8. If α,β,γ are the roots of the equation $x^3-px^2+qx-r=0$, find the value of $\sum\alpha^2$.
9. State Descarte's rule of signs and apply it to prove that the equation $x^3+2x+3=0$ has one negative and two imaginary roots.
10. Explain the term normal form with examples.
11. Define Eigen value and eigen vector.
12. Evaluate the eigen values of $\begin{bmatrix}1&0&0\\0&2&0\\0&0&3\end{bmatrix}$.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Evaluate $grad\phi$ when$\phi=3x^2y-y^3z^2$ at the point (1,-2,-1)
14. Derive the expression$\nabla\left(\frac{\phi_1}{\phi_2}\right)=\frac{\phi_2\nabla\phi_1-\phi_1\nabla\phi_2}{{\phi^2}_2}$.
15. Evaluate the line integrals$\int\limits_c\left(x^2+xy\right)dx+\left(x^2+y^2\right)dy$ where C is the square bounded by the lines $x=\pm1\ and\ y=\pm1.$
16. Evaluate $\int\limits_c\left(2x^2y+y+z^2\right)i+2\left(1+yz^2\right)j+\left(2z+3y^2z^2\right)k.\,d\bar{r}$ along the curve $c: y^2+z^2=a^2, x=0.$
17. Solve $x^3-9x+28=0$ using Cardan's method.
18. Solve the equation $4x^4-8x^3+7x^2+2x-2=0$ given that one root of the equation is $1+i.$

19. Test for consistancy and if consistant solve
$x+2y-z=3, 3x-y+2z=1, 2x-2y+3z=2, x-y+z=-1$

20. Use Gauss Jordan method to find the inverse of $\begin{bmatrix} 2 & 1 & -1 & 2 \\ 1 & 3 & 2 & -3 \\ -1 & 2 & 1 & -1 \\ 2 & -3 & -1 & 4 \end{bmatrix}$.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. a) Prove that $\nabla^2\left(r^n\right)=n\left(n+1\right)r^{n-2}$
b)If $\bar{v}=3x^2y^2z^4i\ +2x^3yz^4j\ +4x^3y^2z^3k\ show\ that\ \bar{v}$ is a conservative field and find its scalar potential.

22. Verify Green's theorem in the plane for $\oint_c\left(3x^2-8y^2\right)dx+(4y-6xy)dy$where c is the boundary of the region bounded by $(a)y=\sqrt{x}, y=x^2$
$(b)x=0, y=o, x+y=1$

23. (a)Show that if α is an r - multiple root of $f(x)=0$, then α is an $r-1$ multiple root of $f'(x)=0$
(b)Solve $x^5-x^3+4x^2-3x+2=0$ given that it has multiple roots.

24. Investigate the values of λ and μ so that the equation $2x+3y+5z=9$, $7x+3y-2z=8$, $2x+3y+\lambda z=\mu$ have
1) no solution 2)a unique solution 3)an infinite number of solution.

(10 x 3 = 30)