Reg. No.. Name.. **21U616**

B. Sc. DEGREE END SEMESTER EXAMINATION - APRIL 2021

SEMESTER **–6: MATHEMATICS (CORE COURSE)**

COURSE: **15U6CRMAT10: COMPLEX ANALYSIS**

(Common for Regular 2018 admission & Improvement 2017/Supplementary 2017/2016 /2015/2014 admissions)

Time: Three Hours Max Marks: 75

PART A

Answer All questions. Each question carries 1 mark.

1. Define continuity of a function $f(z)$ at z_0 .
2. Write the function $f(z) = z^2$ in the form $u(x, y) + i\, v(x, y)$.
3. Find the principal value of $(-i)^i$.
4. State Liouville's theorem.
5. Apply Cauchy-Goursat theorem to show that $\int_C \ tanz\, dz = 0$ where C is the unit circle $|z| = 1$.
6. Find the nature of the singularity of $f(z) = \frac{1-cosz}{z^2}$.
7. Evaluate $\int_1^2 (1 - it)^2\ dt$.
8. Write the Maclaurin series expansion of $f(z) = \frac{1}{2-z}$.
9. Find the residue at $z = 0$ of $f(z) = \frac{1}{z+z^2}$.
10. Find all isolated singularities of $f(z) = \frac{z+1}{z^3(z^2+1)}$. (1 x 10 = 10)

PART B

Answer any Eight questions. Each question carries 2 marks.

11. Verify Cauchy Riemann equation for the function $f(z) = z^3$.
12. Show that $f'(z)$ does not exist at any point if $(z) = \bar{z}$.
13. Show that $Log(-1 + i)^2 \neq 2\, Log\ (-1 + i)$.
14. Evaluate $\int_C \frac{z^2+1}{z^2-1}\ dz$ if C is the circle of unit radius with centre at $z = -1$.
15. Evaluate $\int_C \ z^2\, dz$ where C is the straight line segment joining the origin to the point 2+i.
16. Evaluate $\int_C \frac{z+2}{z}\, dz$ where C is the semi circle $z = 2e^{i\theta}, \pi \leq \theta \leq 2\pi$.
17. State Laurent's theorem.
18. Represent the function $f(z) = \frac{z+1}{z-1}$ by its Maclaurin's series and state where the representation is valid.
19. What is the nature of the singularity of the function $f(z) = sin\left(\frac{1}{1-z}\right)$ at $z = 1$.
20. Find the residue of $f(z) = tanz$ at $z = \frac{\pi}{2}$. (2 x 8 = 16)

PART C

Answer any Five questions. Each question carries 5 marks.

21. Show that $u(x, y) = 2x\,(1 - y)$ is harmonic and find its harmonic conjugate.
22. Show that an analytic function $f(z) = u + iv$ is constant if (i) its real part is constant (ii) its modulus is constant.
23. State and prove Cauchy's integral formula.
24. Let C be the arc of the circle $|z| = 2$ that lies in the first quadrant. Without evaluating the integral, show that $\left|\int_C \frac{dz}{z^2+1}\right| \le \frac{\pi}{3}$.
25. Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in a Laurent's series valid for (i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$.
26. State and prove Cauchy's residue theorem.
27. Evaluate $\int_C \frac{dz}{z^3(z+4)}$ where $C: |z| = 2$. (5 x 5 = 25)

PART D

Answer any Two questions. Each question carries 12 marks.

28. State and prove the necessary and sufficient condition for $f(z) = u + i\,v$ is analytic.
29. (i) State and prove fundamental theorem of algebra.
    (ii) State and prove maximum modulus principle.
30. State and prove Taylor's theorem.
31. Evaluate $\int_0^{2\pi} \frac{d\theta}{a+b\,cos\theta}$, $a > b > 0$. (12 x 2 = 24)

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