Reg. No………………………………………….. Name………………………………………………. **21U604**

B.Sc. DEGREE END SEMESTER EXAMINATION - APRIL 2021

SEMESTER –6: MATHEMATICS (COMMON FOR MATHEMATICS/ COMPUTER APPLICATIONS)

COURSE: 15U6CRMAT9-15U6CRCMT7: REAL ANALYSIS

(Common for Regular 2018 Admission & Improvement 2017/Supplementary 2017/2016/2015/2014 Admissions)

Time: Three Hours Max Marks: 75

PART A

Answer All Questions. Each Question has 1 Mark.

1. State Raabe's test for checking convergence of an infinite series.
2. Give an example of a diverging series $\sum U_n$ with $\lim\limits_{n\to\infty}\left(\frac{u_n}{u_{n+1}}\right)=1$
3. Define conditional convergence of an infinite series.
4. Give an example of a function which is discontinuous at every real numbers.
5. If f and g are functions continuous at a point c, then prove that (f + g) is continuous at c.
6. Is the function f(x) = 2x + 3 is integrable on [2, 3] ? Justify
7. Define lower Darboux sum of a bounded function f over [a,b].
8. If P^* is a refinement of partition P, then show that $U(P^*, f) \leq U(P, f)$.
9. Define pointwise convergence of a sequence of functions.
10. Show that $\sum \frac{x^n a_n}{1+x^n}$ converges uniformly in [0, 1] if $\sum a_n$ converges. (1 x 10 = 10)

PART B

Answer any Eight. Each Question has 2 Marks

11. Investigate the behaviour of the series whose n^{th} term is $\cos(1/n^3)$.
12. Let $\sum u_n$ be a positive term series such that $\lim\limits_{n\to\infty} n\left(\frac{u_n}{u_{n+1}}-1\right)=l$,

    then prove that the series converges if $l>1$.
13. Prove or disprove : Every absolutely convergent series is convergent.
14. Show that f(x) = sin x is uniformly continuous on [0, ∞)
15. Discuss the continuity of the function f(x) = [x] where [x] is the greatest integer less than or equal to x
16. Show that a constant function k is integrable and $\int_a^b k\,dx = k(b-a)$.

17. If f_1 and f_2 are bounded and integrable functions on [a,b], then show that their sum $f = f_1 + f_2$ is also integrable on [a,b] .
18. If a function f is continuous on [a,b], then show that it is integrable on [a,b].
19. State and prove Abel's test for uniform convergence of a series of functions.
20. Show that the sequence of functions $\{f_n(x)\}$ where $f_n(x) = nxe^{-nx^2}, \quad x \geq 0$ is not uniformly convergent on [0, k], k > 0. (2 x 8 = 16)

PART C

Answer any Five. Each Question has 5 Marks.

21. Check the convergence of the series whose n^{th} term is given by $\frac{3.6.9...3n}{7.10.13...(3n+4)}x^n$ where $x > 0$
22. State and prove Cauchy's general principle of convergence for infinite series
23. If a function f is continuous at an interior point c of [a, b] and f(c) $\neq$ 0 then prove the existence of a neighbourhood of c where f(x) has the same sign as f(c).
24. Prove that a function which is continuous on a closed interval is also uniformly continuous on that interval.
25. State and prove second fundamental theorem of integral calculus
26. Prove that a bounded function having a finite number of points of discontinuity on [a,b] is integrable on [a,b].
27. State and prove Weierstrass M- test for uniform convergence of a series of functions.

(5 x 5 = 25)

PART D

Answer any Two. Each Question has 12 Marks.

28. Prove that the series $\sum \frac{1}{n^p}$ is convergent if and only if $p > 1$
29. (a) If a function f is continuous on [a,b] and f(a).f(b) < 0, then prove that there exist at least one point c in (a, b) such that f(c) = 0.

    (b) Show that the function f(x) defined on R by $f(x) = \begin{cases} x, & \text{if } x \text{ is irrational} \\ -x, & \text{if } x \text{ is rational} \end{cases}$ is continuous only at $x = 0$.
30. (a) State and prove a necessary and sufficient condition for the integrability of a bounded function on a closed interval.

    (b) State and prove fundamental theorem of integral calculus.
31. (a) State and prove Cauchy's criterion for uniform convergence of a sequence of functions

    (b) Show that the sequence of functions $\{f_n\}$ where $f_n(x) = \frac{x}{1+nx^2}$, x being real converges uniformly on any closed interval. (12 x 2 = 24)