Reg. No ..... Name **21U248**

B Sc DEGREE END SEMESTER EXAMINATION - JULY 2021

SEMESTER 2 : COMPLEMENTARY MATHEMATICS FOR B Sc CHEMISTRY /PHYSICS

COURSE : **19U2CPMAT2 : CALCULUS -2 AND NUMERICAL ANALYSIS**

(For Regular - 2020 Admission and Supplementary - 2019 Admission)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. If $\overrightarrow{r} = xi + yj + zk$, show that $\nabla r^n = n^{n-2}\overrightarrow{r}$
2. If $\overrightarrow{r} = xi + yj + zk$, show that $\nabla\left(\overrightarrow{a}.\ \overrightarrow{r}\right) = \overrightarrow{a}$, where $\overrightarrow{a}$ is a constant vector
3. Show that $\nabla \times \left(\overrightarrow{A} + \overrightarrow{B}\right) = \nabla \times \overrightarrow{A} + \nabla \times \overrightarrow{B}$
4. If $\overrightarrow{r} = xi + yj + zk$, show that $grad\left(\frac{1}{r}\right) = \frac{-\overrightarrow{r}}{r^3}$
5. For any closed surface S, prove that $\iint_S curl\,\overrightarrow{F}\,.\,\hat{n}\,dS = 0.$
6. Evaluate $\Delta tan^{-1}ax$ with interval of difference as unity.
7. Prove that $\nabla = 1 - E^{-1}$
8. Define Simpson's one third rule.
9. Define Simpson's three eighth rule.
10. Given first two approximations a and b , what is the next approximation using bisection method?
11. Define the general form of nth root in Newton's method.
12. Write the condition for convergence of Newton-Raphson method.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$
14. Explain the physical interpretation of divergence
15. Use Green's theorem in the plane to evaluate the integral $\oint_C (2x^2 - y^2)\,dx + (x^2 + y^2)\,dy$ where C is the boundary in the xy- plane of the area enclosed by the x- axis and the semicircle $x^2 + y^2 = 1$ in the upper half xy - plane.
16. If $\overrightarrow{A} = 2xz\hat{i} - x\hat{j} + y^2\hat{k}$, evaluate $\iiint_V \overrightarrow{A}\ dV$, where V is the region bounded by the surface $x = 0, y = 0, x = 2, y = 6, z = x^2, z = 4.$
17. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's three - eighth rule with h = 1/6
18. Prove that $\Delta = \frac{1}{2} \delta ^ 2 + \delta \sqrt {1+\frac{1}{4}\delta ^ 2}$
19. Find the real root of the equation $x^4 - x - 9 = 0$ by Newton-Raphson method correct to three place of decimal.
20. Find the real root of the equatioon $xe^x - 3 = 0$ by Regula-Falssi method, correct to three decimal places.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. a. Prove that the vector $f\left(r\right)\overrightarrow{r}$ is irrotational

    b. Prove that $\nabla^2 f\left(r\right) = f^{''}\left(r\right) + \frac{2}{r}f'\left(r\right)$

22. Verify Stoke's theorem for $\overrightarrow{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ over the upper half surface of $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy- plane.

23. Using Newton's interpolation formula and the given table of values find the value of y at x = 1.4.

x	1.1	1.3	1.5	1.7	1.9
y	0.21	0.69	1.25	1.89	2.61

24. Using Horner's method find the root of $x^3 + 9x^2 - 18 = 0$, correct to two decimal places.

(10 x 3 = 30)