Reg. No .....

Name .....

21U225

B Sc DEGREE END SEMESTER EXAMINATION - JULY 2021

SEMESTER 2 : MATHEMATICS (CORE COURSE)

COURSE : **19U2CRMAT2 : ADVANCED CALCULUS AND TRIGONOMETRY**

(For Regular - 2020 Admission and Supplementary - 2019 Admission)

Time : Three Hours

Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Expand $\cos x$ by Maclaurin's series.
2. Expand $\log \sin x$ in powers of $x - 2$.
3. Find the n^{th} derivative of $y = (ax + b)^m$.
4. Find the circumference of a circle of radius r using parametric forms.
5. Find $y'(x),\ y''(x)$ without eliminating the parameter for the curve $x = sect,\ y = \tan t\ ;\ t = \frac{\pi}{4}$.
6. Sketch the graph of $r = \theta, \theta \geq 0$ in polar coordinates by plotting points.
7. Find the arc length of the spiral $r = e^{\theta}$ between $\theta = 0$ and $\theta = \pi$.
8. Separate into its real and imaginary parts the expression $\cosh(\alpha + \beta i)$.
9. Prove that tanh u = $\sin\theta$
10. Define area using double integral.
11. Write the parametric equation of the paraboloid z = 4 — x^2 — y^2
12. Define simple polar region with example

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Find the equation of the circle of curvature at the point $(0, b)$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
14. Find all the asymptotes of the curve $y^3 - 6xy^2 + 11x^2y - 6x^3 + x + y = 0$.
15. Find the points on the cardioid $r = 1 - \cos\theta$ at which there is a horizontal tangent line , a vertical tangent line , or a singular point.
16. Find the area of the region in the first quadrant that is within the cardioid $r = 1 - \cos\theta$.
17. Sum the series $\sin\alpha + c\sin(\alpha + \beta) + \frac{c^2}{2!}\sin(\alpha + 2\beta) + \cdots$ inf
18. Show that $\cos h^{-1}x\ =\ \log\ \left[x\ +\ \sqrt{x^2 - 1}\right]$, when x is real.
19. Use double integration to find the area enclosed by the curves y = sin x and y = cos x for π/4≤x≤π/2
20. Find the value of the $\iint xydA$ over the region enclosed between $y\ = \frac{x}{2},\ y\ =\ \sqrt{x}\ ,\ x = 2\ and\ x = 4$

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Prove Leibnitz theorem. If $y = \left(x^2 - 1\right)^n$, prove that $\left(x^2 - 1\right)y_{n+2} + 2xy_{n+1} - n\left(n + 1\right)y_n = 0$.

22. Sum to n terms and to infinity the series $1 + a\ \cos\ \theta\ + a^2\ \cos\ 2\theta\ + a^3\ \cos\ 3\theta + \ldots, \quad |a| < 1$

23. Define area and volume using double integrals. Evaluate $\int\int \sin\theta\, dA$ where the region is the first quadrant that is evaluated outside the circle r = 2 and inside the cardiod r = 2(1+cosθ).

24. Find the volume of the solid enclosed between the paraboloids $z\ =\ 5x^2 + 5y^2\ and\ z\ =\ 6\ - 7x^2 - y^2$.

(10 x 3 = 30)