Reg. No…………………………………………. Name…………………………………………… **21U224**

B. Sc. DEGREE END SEMESTER EXAMINATION JULY - 2021

SEMESTER **– 2: MATHEMATICS (CORE COURSE FOR MATHEMATICS AND COMPUTER APPLICATIONS)**

COURSE: **15U2CRMAT2, 15U2CRCMT2: ANALYTIC GEOMETRY, TRIGONOMETRY AND MATRICES**

(Common for supplementary 2018/2017/2016/2015/2014 admissions)

Time: Three Hours Max. Marks: 75

PART A

Answer all questions. Each question carries 1 mark

1. Write the equation of the tangent to the ellipse at the point (x_1, y_1).
2. Show that $y = \pm\frac{b}{a}x$ are the asymptotes of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
3. Represent a point on the circle with centre at *a* and radius *r* using a single parameter.
4. Define the term latus rectum.
5. Write the polar equation of a parabola.
6. Prove that $\sin(ix) = i\, sinhx$.
7. Separate sin(a + i P) into real and imaginary parts.
8. Prove that $\cosh\Theta = \cos(i\,\Theta)$
9. If A is a 3 × 3 matrix of rank 3, what can you say about the solutions of the linear system AX = 0?
10. Write the necessary and sufficient condition for the system $AX = B$ is consistent

(1 x 10 = 10)

PART B

Answer any eight questions. Each question carries 2 marks.

11. Find the condition for the line $y = mx + c$ to be a tangent to the parabola $y^2 = 4ax$.
12. Find the pole of the line $lx + my + n = 0$ with respect to $x^2 + y^2 = a^2$.
13. Show that the sum of the squares of two conjugate semi diameters of an ellipse is constant.
14. Prove that the polar of a point $(x_{1,}y_1)$ with respect to the parabola $y^2 = 4ax$ is $yy_1 = 2a(x+x_1)$
15. What is the polar equation of a conic?
16. Find the real and imaginary parts of $\tan(\alpha + \beta i)$.
17. Prove that $cosh^{-1} x = \log(x + \sqrt{x^2 - 1})$.
18. Show that the eigen values of a triangular matrix are its entries on the main diagonal.
19. Give the definition of elementary matrix. Find the elementary matrix $H_{23}(4)$ obtained from I_3
20. Separate $\log(\alpha + i\beta)$ into real and imaginary parts.

Type equation here.

(2 x 8 = 16)

PART C

Answer any five questions. Each question carries 5 marks

21. Show that the locus of midpoints of chords of a parabola which subtend a right angle at the vertex is another parabola of half the latus rectum of the original parabola.
22. Derive the condition for two circles to cut one another orthogonally
23. Find the linear factor of $x^8 + 1$.
24. Prove that the tangents at the ends of a focal chord of the parabola $y^2 = 4ax$ intersect at right angles on the directrix.
25. Find the factorization of $x^n - 1$ when n is even.
26. Reduce the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ to its normal form hence find its rank.
27. Find the equation of the pair of tangents from $(x_1\ y_1)$ to the parabola $y^2 = 4ax$

(5 x 5 = 25)

PART D

Answer any two questions. Each question carries 12 marks.

28. Find the equation of the pair of tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from the point (x_1, y_1).
29. A normal chord to an ellipse makes an angle of 45^0 with the axis. Prove that the square of its length is equal to $\frac{32a^4b^4}{(a^2+b^2)^3}$.
30. Prove that $\sin\theta = \theta\prod_{r=1}^{\infty}\left(1 - \frac{\theta^2}{r^2\pi^2}\right)$.
31. Find the characteristic equation of the matrix A = $\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

(12 x 2 = 24)

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