Reg. No .................... Name .............................. **21U422**

**B Sc DEGREE END SEMESTER EXAMINATION - JULY 2021**

**SEMESTER 4 : MATHEMATICS**

COURSE : **19U4CRMAT4 ANALYTIC GEOMETRY NUMERICAL METHODS AND NUMBER THOERY**

*(For Regular - 2019 Admission)*

Time : Three Hours Max. Marks: 75

**PART A**

**Answer any 10 (2 marks each)**

1. The distance of a point from the centre of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ is $\frac{5}{\sqrt{2}}$. Find the eccentric angle of this point.
2. Find the equation of the bisectors of the angles between the lines $x^2-4xy-y^2=0.$
3. For what point of the parabola $y^2=18x$ is the ordinate equal to three times the abscissa?
4. Find the equation of the normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point $(x_1,y_1)$.
5. Find the polar equation of the line passing through $(r_1,\theta_1)$ and $(r_2,\theta_2)$.
6. Find the points on the conic $\frac{9}{r}=2+\sqrt{2}\cos\theta$ whose radius vector is 3.
7. Transform the equation to polar form $x^2+y^2-ax-by=0.$
8. Use Newton-Raphson method to find a root of the equation $x=e^{-x}$.
9. Evaluate $f(2)$ where $f(x)=\log x+x-\cos x.$
10. Explain complete system of residues.
11. Define quadratic congruence.
12. Show that if $a\equiv b(\mod n)$ and $b\equiv c(\mod n)$, then $a\equiv c(\mod n)$.

**(2 x 10 = 20)**

**PART B**

**Answer any 5 (5 marks each)**

13. Derive the standard equation of hyperbola.
14. Show that the equation of the ellipse whose axes ore of lengths $8$ and $6$ and equations are $4x+3y-2=0$ and $3x-4y+1=0$ is $\frac{(3x-4y+1)^2}{64}+\frac{(4x+3y-2)^2}{36}=25.$
15. Prove that the distances of two points $(x_1,y_1)$ and $(x_2,y_2)$ from the centre of a circle are proportional to the distance of each from the polar of the other.
16. Find the equation of pair of asymptotes to the general equation of hyperbola.
17. Use Newton-Raphson method to find a root correct to three decimal places, of the equation $\sin x=x/2$, given that the root lies between $\pi/2$ and $\pi$.
18. Obtain a root, correct to three decimal places, of the equation $x^3+x^2-1=0$ using the bisection method.
19. Let $n=p_1p_2\dots p_r$ be a composite square-free integer, where the $p_i$ are distinct primes. If $p_i-1|n-1$ for $i=1,2,\dots,r$, then prove that $n$ is an absolute pseudoprime.
20. If the integer $n>1$ has the prime factorization $n=p_1^{k_1}p_2^{k_2}\cdots p_r^{k_r}$, then prove that $\phi(n)=n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)\cdots\left(1-\frac{1}{p_r}\right).$

**(5 x 5 = 25)**

**PART C**

**Answer any 3 (10 marks each)**

21. Show that the equation $7x^2 - 48xy - 7y^2 - 20x + 140y + 300 = 0$ represents a hyperbola and find its canonical equation.
22. Show that four normals can be drawn to a hyperbola through a given point and the locus of the feet of these normals is a rectangular hyperbola.
23. Explain briefly the method of iteration to compute a real root of the equation $f(x) = 0$, stating the condition of convergence of the sequence of approximations. Give a graphical representation of the method.
24. Prove that the quadratic congruence $x^2 + 1 \equiv 0 \pmod{p}$, where $p$ is an oddd prime, has a solution if and only if $p \equiv 1 \pmod{4}$.

**(10 x 3 = 30)**