Reg. No………………………………………….   Name…………………………………………   **21U421-S**

B. Sc. DEGREE END SEMESTER EXAMINATION – JULY 2021

SEMESTER **– 4: MATHEMATICS (CORE FOR MATHS AND COMPUTER APPLICATIONS)**

COURSE: **15U4CRMAT04-15U4CRCMT04, VECTOR CALCULUS, THEORY OF EQUATIONS AND NUMERICAL METHODS**

(Common for Improvement 2018 admission / Supplementary 2018/2017/2016/2015 admissions)

Time: Three Hours   Max. Marks: 75

PART A

Answer all questions. Each question carries 1 mark

1. Find the angle between the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$
2. Show that the curvature of a straight line is Zero
3. Define Radius of Curvature
4. Define work over a smooth curve
5. State Gauss-Divergence theorem
6. Find the divergence of $F(x, y) = (x^2 - y)i + (xy - y^2)j$
7. Define Reciprocal equation
8. State Descartes' Rule of Signs
9. Write the Newton-Raphson formula.
10. State the sufficient conditions for the convergence of the sequence defined by $x_{n+1} = \emptyset(x_n)$ in iteration method   (1 × 10 = 10)

PART B

Answer any eight questions. Each question carries 2 marks

11. Find a plane through $P_0(2, 1, -1)$ and perpendicular to the line of intersection of the planes $2x + y - z = 3,\ x + 2y + z = 2$
12. Find the derivative of the function $f(x, y, z) = 3e^x \cos(yz)$ at $P_0(0, 0, 0)$ in the direction of $$A = 2i + j - 2k$$
13. If $r(t) = (3cost)i + (3sint)j + t^2k$ is the position vector of the particle in space at time t. Find the times when the acceleration is orthogonal to its velocity
14. Find the circulation of the field $F = (x - y)i + xj$ around the circle $r(t) = (cost)i + (sint)j\ ;\ 0 \le t \le 2\pi$
15. Find the line integral of $f(x, y, z) = x - 3y^2 + z$ over the line segment C joining the origin to the point (1, 1, 1)
16. Find a Potential function for the field $F = (y + z)i + (x + z)j + (x + y)k$
17. If α, β, γ are the roots of $8x^3 - 4x^2 + 6x - 1 = 0$. Find the equation whose roots are $2\alpha + 1, 2\beta + 1,\ 2\gamma + 1$
18. Solve $4x^3 - 24x^2 + 23x + 18 = 0$, given that the roots are in Arithmetic Progression
19. Using the bisection method find a real root of the equation $x^3 - x - 1 = 0$ correct to two decimal places/
20. Explain the method of false position to find the real root of non linear equation $f(x) = 0$

(2 × 8 = 16)

PART C

Answer any Five questions. Each Question carries 5 marks

21. The Surfaces $f(x,y,z) = x^2 + y^2 - 2 = 0$ *and* $g(x,y,z) = x + z - 4 = 0$ meet in an ellipse E. Find parametric equations for the line tangent to E at the point $P_0(1,1,3)$
22. Find the Binormal Vector for the space curve $r(t) = (cost + tsint)i + (sint - tcost)j + 3k$
23. Show that the differential form is exact and then evaluate $\int_{(1,1,1)}^{(2,2,2)} \frac{1}{y}dx + \left(\frac{1}{z} - \frac{x}{y^2}\right)dy - \frac{y}{z^2}dz$
24. Find the area of the surface cut from the bottom of the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 0$
25. Solve the equation $3x^5 - 10x^4 - 3x^3 - 3x^2 - 10x + 3 = 0$
26. Solve $2x^4 - 15x^3 + 35x^2 - 30x + 8 = 0$, whose roots are in G.P
27. Find a real root of $x^3 + 3x^2 - 3 = 0$, using the Newton –Raphson method

(5 × 5 = 25)

PART D

Answer any two questions. Each question carries 12 marks

28. Without finding T and N write the acceleration of the motion $r(t) = (e^t cost)i + (e^t sint)j + \sqrt{2}e^t k$ at $t = 0$ in the form $a = a_T T + a_N N$
29. State Stokes theorem. Verify Stokes Equation for a Hemisphere $S: x^2 + y^2 + z^2 = 9, z \geq 0$, its bounding circle $C: x^2 + y^2 = 9, z = 0$, and the field $F = yi - xj$
30. Explain the Iteration Method. Use the method of iteration to find a positive root of the equation $e^{-x} = 10x$ correct to four decimal places.
31. Solve the equation $x^4 + 3x^3 + x^2 - 2 = 0$

(12 × 2 = 24)

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