Reg. No:.............................. Name:................................. Q. Code: **U122**.

BSC DEGREE END SEMESTER EXAMINATION

SEMESTER-1: MATHEMATICS CORE COURSE FOR BSC MATHEMATICS/ BSC COMPUTER APPL.

COURSE - 15U1CRMAT1-15U1CRCMT1: FOUNDATION OF MATHEMATICS

Time : Three Hours Max. Marks:75

Part A

Answer *all* questions. Each question carries 1 mark.

1. Find the power set of $\{\{\emptyset, a\}\}$.
2. Find the number of elements in $A \times B \times C$, where $A = \{1,2,4\}$ and $B = C = \{0\}$.
3. Let $M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ be a matrix representing a relation R. Find the matrix representing the relation R^2.
4. How many relations are there on the set $\{a,b,c,d\}$?
5. Is the relation $R = \{(1,1),(1,2),(2,3),(1,3),(4,4)\}$ symmetric? Justify your
6. What is the negation of the proposition '*2+1=4*'
7. What is the truth value of the compound proposition '21 < 32 and 2+3=6'
8. Identify the law. $\neg(p \wedge q) \equiv \neg p \vee \neg q$.
9. Is 28 a perfect number? justify your Answer.
10. What are amicable numbers?

(10 X 1 = 10)

Part B

Answer any *eight* questions. Each question carries 2 marks.

11. Prove or disprove that d*x* + *y*e = d*x*e + d*y*e for all real numbers *x* and *y*.
12. If $f(x) = 2x + 3$ and $g(x) = 3x + 2$, find $(f \circ g)(x)$.

13. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$, where $A_i = \{i, i+1, i+2\}$, for $i = 2,3,4,...$

14. List the ordered pairs in the relation R from $A = \{0,1,2,3,4\}$ to $B = \{0,1,2,3\}$, where $(a,b) \in R$ if and only if $a + b = 4$.

15. Is the relation ≤ defined on R is anti-symmetric? Explain.

16. Find the bitwise OR of 0110110110 and 1100011101. Also, find the length of the resultant bit string.

17. Prove that $p \vee (p \wedge q) \equiv p$, for any two propositions p and q.

18. If the domain of $Q(x,y,z)$ consists of (x,y,z), where $x \in \{0,1,2\}, y \in \{0,1\}$ and $z \in \{0,1\}$, then rewrite the proposition $\forall y Q(0,y,0)$ using only negations, disjunctions and/or conjunctions.

19. Write the gcd of 81 and 110 as the linear combination of the two integers.

20. What is Euler's totient φ? Calculate $\varphi(500)$.

(8 X 2 = 16)

Part C

Answer any *five* questions.Each question carries 5 marks.

21. If a and r are real numbers, r 6= 0, prove that the value of $\sum_{j=0}^{n} ar^j$ is $\frac{ar^{n+1} - a}{r - 1}$, when r 6= 1.

22. Let $R_1 = \{(1,2),(2,3),(3,4)\}$ and $R_2 = \{(1,1),(1,2),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(3,4)\}$ be relations from $\{1,2,3\}$ to $\{1,2,3,4\}$. Then find (a) $R_1 \cup R_2$ (b) $R_1 \cap R_2$ (c) $R_1 - R_2$ (d) $R_2 - R_1$ and (e) $R_1 \oplus R_2$

23. Draw the directed graph of the relation $R = \{(1,1),(1,3),(2,1),(2,3),(2,4),(3,1),(3,2),(4,1)\}$ on the set $\{1,2,3,4\}$. Using the graph, explain whether the relation R is reflexive, symmetric or anti-symmetric.

24. Show that $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$.

25. Give a logical proof for $\neg \forall x P(x) \equiv \exists x \neg P(x)$.

26. Show that $n(n + 1)(2n + 1)$ is divisible by 6.

27. Show that $3^{2n} + 24n - 1$ is divisible by 32.

(5 X 5 = 25)

Part D

Each question carries 12 marks.

28. (a) Prove that if B is countable subset of an uncountable set A, then $A - B$ is uncountable.

(b) Show that the set of all irrational numbers is uncountable

29. Prove that a relation R on a set A is transitive if and only if $R^n \subseteq R$ for $n = 1,2,3,\ldots$.

30. Show that if n is an integer and $n^3 + 5$ is odd, then n is even using

(a) a proof by contraposition

(b) a proof by contradiction

31. State and prove Little Fermat's Theorem.

(2 X 12 = 24)

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