Reg. No .... Name **20U156**

B. Sc DEGREE END SEMESTER EXAMINATION - OCT. 2020 : FEBRUARY 2021

SEMESTER 1 : COMPLEMENTARY MATHEMATICS FOR B Sc PHYSICS & CHEMISTRY

COURSE : **19U1CPMAT1 : CALCULUS-1**

(For Regular - 2020 Admission & Improvement / Supplementary - 2019 Admission)

Time : Three Hours Max. Marks: 75

PART A

Answer any 10 (2 marks each)

1. Draw a tree diagram and write a Chain Rule formula for $\frac{\partial y}{\partial r}$ and $\frac{\partial y}{\partial s}$ for $y = f(u), u = g(r, s)$.
2. Draw a tree diagram and write a Chain Rule formula for $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ for $w = f(x, y), x = g(r), y = h(s)$.
3. State the second derivative test for local extreme values of $f(x, y)$.
4. Let f be continuous on the symmetric interval $[-a, a]$. Show that $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$, if f is even.
5. A pyramid $3\,m$ high has a square base that is $3\,m$ on a side. The cross-section of the pyramid perpendicular to the altitude $x\,m$ down from the vertex is a square $x\,m$ on a side. Find the volume of the pyramid.
6. The region between the curve $y = \sqrt{x},\ 0 \le x \le 4$ and the x-axis is revolved about the x-axis to generate a solid. Find its volume.
7. State first derivative theorem for local extreme values.
8. Verify Rolle's theorem for the function $f(x) = \frac{x^3}{3} - 3x$ in the interval $[-3, 3]$
9. State first derivative test for monotonic functions.
10. Verify mean value theorem for the function $f(x) = x^2 + 2x - 1$, in the interval $[0, 1]$
11. Find the limits of integration for integrating $f(r, \theta)$ over the region R that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$.
12. Write the formula for finding area of a closed and bounded plane region using double integrals.

(2 x 10 = 20)

PART B

Answer any 5 (5 marks each)

13. Show that the function $f(x, y) = y^2 - x^2$ has a saddle point at the origin.
14. State a formula using partial derivatives for determining $\frac{dy}{dx}$ from the equation $F(x, y) = 0$ which defines y as a differentiable function of x. Using this formula find $\frac{dy}{dx}$ if $xy + y^2 - 3x - 3 = 0$ at the point $(-1, 1)$.
15. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$ by integrating with respect to y.
16. Find the volume of the solid generated by revolving each region enclosed by the triangle with vertices $(1, 0), (2, 1)$, and $(1, 1)$ about the y-axis.

17. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x},\ 1 \leq x \leq 2$ about the x-axis.
18. Find the intervals on which the function $h(x) = 2x^3 - 18x$ is increasing and decreasing.
19. Show that the equation $x^3 + 3x + 1 = 0$ has exactly one real solution.
20. Sketch the region of integration and evaluate the integral $\int_{-1}^{0}\int_{-1}^{1}(x+y+1)dy\,dx$.

(5 x 5 = 25)

PART C

Answer any 3 (10 marks each)

21. Find the points on the curve $xy^2 = 54$ nearest the origin.
22. Find the area of the region in the first quadrant bounded by the line $y = x$, the line $x = 2$, the curve $y = 1/x^2$ and the x-axis.
23. Find the critical points of $f(x) = \dfrac{x^3}{3x^2+1}$ and identify the intervals on which f is increasing and decreasing. Find the function's local and absolute extreme values.
24. A thin plate covers the triangular region bounded by the x - axis and the lines $x = 1$ and $y = 2x$ in the first quadrant. The plate's density at the point (x, y) is $\delta(x, y) = 6x + 6y + 6$. Find the plate's mass, first moments, and center of mass about the coordinate axes. Also find the moments of inertia and radii of gyration about the coordinate axes and the origin.

(10 x 3 = 30)