Reg. No ..... Name **17P3631**

MSc DEGREE END SEMESTER EXAMINATION- OCTOBER-NOVEMBER 2017

SEMESTER 3 : MATHEMATICS

COURSE : **16P3MATT13 ; GRAPH THEORY**

(For Regular - 2016 admission)

Time : Three Hours Max. Marks: 75

Section A

Answer any 10 (1.5 marks each)

1. Show that the sum of the degrees of the vertices of a graph is equal to twice the number of its edges.
2. Let G be a 2-connected graph and u and v be vertices of G. Let P be a $u-v$ path in G.Is it necessarily true that there exists another $u-v$ path in G such that P and Q are internally disjoint? Justify your answer.
3. Define diameter of a graph, eccentricity of a vertex of a graph and radius of a graph.
4. Give an example of a tree with two central vertices, one of which is also a centroidal vertex.
5. Give an example of a graph which has its radius equal to its diameter.
6. Draw the Petersen graph
7. Define an Eulerian graph.Give examples of Eulerian and non-Eulerian graphs.
8. Determine $\chi(P)$, where P is the Petersen graph.
9. Determine $\chi'(K_4)$
10. Let f be a plane graph and f be a face of G. Show that there exists a plane embedding of G in which f is the exterior face.

10 x 1.5 (15)

Section B

Answer any 4 (5 marks each)

11. Show that a tournament is diconnected if and only if it has a spanning direct cycle.
12. (a) Show that a tree with at least two vertices contains at least two pendant vertices.
    (b) Show that if $\delta(G) \geq 2$, then G contains a cycle.
13. Show that a subset S of V,the vertex set of a graph is independent if and only if $V-S$ is a covering of G.By means of an example, show that the edge analogue of this theorem need not be true.
14. Briefly describe Hamilton's "Around the World Game" and its significance.

15. Prove that the Petersen graph P is not Hamiltonian.

16. Show that K_5 is nonplanar.

4 x 5 (20)

Section C
Answer any 4 (10 marks each)

17.1. (a) Show that the connectivity and edge connectivity of a simple cubic graph G are equal.
(b) Give examples of cubic graphs G_1, G_2 and G_3 with $\kappa(G_1) = 1, \kappa(G_2) = 2, \kappa(G_3) = 3$.
OR

2. State and prove Whitney's theorem on 2- connected graphs.

18.1. Explain Dijkstra's Algorithm using an example.
OR

2. Show that every tree has a center consisting of either a single vertex or two adjacent vertices.

19.1. Show that a graph G is Eulerian if and only if it has an odd number of cycle decompositions.
OR

2. Show that for every positive integer k, there exists a triangle-free graph with chromatic number k.

20.1. (a) Explain the terms plane embedding and spherical embedding.
(b) Explain stereographic projection.
(c) Show that a graph is planar if and only if it is embeddable on a sphere.
OR

2. Show that $\chi'(K_n) = \begin{cases} n-1 & \text{if } n \text{ is even,} \\ n & \text{if } n \text{ is odd.} \end{cases}$

4 x 10 (40)