Name:____________________

Reg No:____________________

BSC DEGREE END SEMESTER EXAMINATION OCTOBER 2016

SEMESTER III: MATHEMATICS (COMPLEMENTARY COURSE FOR PHYSICS AND CHEMISTRY)

COURSE: 15U3 CPMAT3 - VECTOR CALCULUS, DIFFERENTIAL EQUATIONS AND ANALYTIC GEOMETRY

Time: 3 hours **Max Marks: 75**

Part A
Each question carries 1 mark
Answer all the questions

1. Define curvature.
2. Define arc length.
3. State Divergence Theorem.
4. Define conservative field.
5. What is the line integral corresponding to the work done along the curve.
6. Define an Exact differential equation.
7. Define a linear differential equation.
8. Solve : $(y\cos x + \sin y + y)dx + (\sin x + x\cos x + x)dy = 0$
9. Write the polar equation for a conic with eccentricity e.
10. Find the centre and foci of the ellipse: $\dfrac{(x-1)^2}{4} + \dfrac{(y+1)^2}{4} = 1$.

Part B
Each question carries 2 marks
Answer any eight.

11. Find the directional derivative of $f(x,y,z) = xy^2 + yz^3$ at $(2,-1,1)$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.
12. Find the gradient, $\nabla\phi$ for $\phi(x,y,z) = \log(x^2+y^2+z^2)$.
13. Integrate $f(x,y,z) = x - 3y^2 + z$ over the line segment C joining the origin to the point $(1,1,1)$.
14. Find the work done by the force field $\vec{F} = (y-x^2)\hat{i} + (z-y^2)\hat{j} + (x-z^2)\hat{k}$ along the curve $\vec{r}(t) = t\hat{i} + t^2\hat{j} + t^3\hat{k}, 0 \le t \le 1$ from $(0,0,0)$ to $(1,1,1)$.
15. Show that $\vec{F} = (2x-3)\hat{i} + -z\hat{j} + \cos z\hat{k}$ is not conservative.
16. Solve: $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0$.
17. Solve: $x^2(y - px) = yp^2$.
18. Find the centre, foci and eccentricity of $9x^2 + 5y^2 - 54x - 40y + 116 = 0$.

19. Obtain the parametric representation of a parabola.

20. Establish the relation between the focus and directrix of a conic.

Part C
Each question carries 5 marks
Answer any five.

21. Find the unit tangent vector of the curve $\vec{r}(t) = 3\cos t\hat{\imath} + 3]\sin t\hat{\jmath} + t^2\hat{k}$.

22. Find the curvature of a circle of radius a.

23. If $\vec{F} = 3xy\hat{\imath} + -y^2\hat{\jmath} + 0\hat{k}$; evaluate $\int_c \vec{F}\cdot d\vec{r}$.

24. Find f such that $\vec{F} = \nabla f$ where $\vec{F} = (e^x\cos y + yz)\hat{\imath} + (xz - e^x\sin y)\hat{\jmath} + (xy+z)\hat{k}$.

25. Solve: $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$.

26. Solve: $p = \sin(y - xp)$.

27. The hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is shifted 2 units to the right to generate the hyperbola $\frac{(x-2)^2}{16} - \frac{y^2}{9} = 1$. Find the centre, focus, vertices and asymptotes of the new hyperbola.

Part D
Each question carries 12 marks
Answer any two.

28. Verify Stokes's theorem for $\vec{F} = (x^2+y^2+z^2)\hat{\imath} + (-2xy)\hat{\jmath} + 0\hat{k}$, taken round the rectangle bounded by the lines $x = \pm a, y = 0, y = b$.

29. Verify the Divergence theorem for $\vec{F} = (x^2 - yz)\hat{\imath} + (y^2 - zx)\hat{\jmath} + (z^2 - xy)\hat{k}$, taken over the rectangular parallelepiped $0 \le x \le a$, $0 \le y \le b$ and $0 \le z \le c$.

30. Solve
    (a) $(5x^4 + 3x^2y^2 - 2xy^3)dx + (2x^3y - 3x^2y^2 - 5y^4)dy = 0$.
    (b) $y + px - x^4p^2$

31. (a) Find all the polar coordinates of the point $P\left(2, \frac{\pi}{6}\right)$.
    (b) Find the centre, eccentricity, focus and directrix of the conic $9x^2 - 16y^2 + 72x - 32y - 16 = 0$.

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Time: 3 hours **Max Marks: 75**

Part A
Each question carries 1 mark
Answer all the questions

1. Define unit tangent vector.
2. Show that the curvature of $\vec{F(t)} = (a+bt)\hat{i} + (c+dt)\hat{j} + (e+ht)\hat{k}$ is zero.
3. State Green's Theorem.
4. What is meant by the path independence for a vector field $\vec{F}$.
5. Define the surface integral over the surfaces.
6. Define Lagrange's differential equation.
7. Define Clairaut's Differential equation.
8. Solve $\frac{dy}{dx} + \frac{y}{x} = x^3 - 3$.
9. Classify the conic sections by using eccentricity.
10. Define a parametric curve.

Part B
Each question carries 2 marks
Answer any eight

11. Find the directional derivative of $f(x,y,z) = xy^2 + yz^3$ at $(2,-1,1)$ in the direction of the vector $1\hat{i} + 2\hat{j} + 2\hat{k}$.
12. Find the unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point $(1,2,-1)$.
13. Evaluate $\int_c \vec{F} \cdot d\vec{r}$ where $\vec{F} = 2\hat{i} + xy\hat{j} + (-y^2)\hat{k}$ along the curve C given by $\vec{r}(t) = t^2\hat{i} + t\hat{j} + \sqrt{t}\hat{k}, 0 \leq t \leq 1$.
14. Find the work done by the force field $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ in moving an object along the curve C parametrized by $\vec{r}(t) = \cos(\pi t)\hat{i} + t^2\hat{j} + \sin(\pi t)\hat{k}, 0 \leq t \leq 1$.
15. Evaluate $\oint_C xydy - y^2dx$, where C is the square cut form the first quadrant by the lines $x = 1$ and $y = 1$.

16. Solve $ye^{xy}dx + (xe^{xy} + 2y)dy = 0$.

17. Solve: $(px + y)^2 = py^2$.

18. Find the centre, foci and eccentricity of the conic $4x^2 - 9y^2 - 8x - 18y - 41 = 0$.

19. Find the polar equation for the circle $x^2 + (y-3)^2 = 9$.

20. Sketch and discuss about the curve cycloid.

Part C
Each question carries 5 marks
Answer any five.

21. Find the arc length of $\vec{r}(t) = \cos t\hat{i} + \sin t\hat{j} + t\hat{k}$ along the path from $t = 0$ to $t = 2\pi$.

22. Find the Binormal vector of $\vec{r}(t) = \cos t\hat{i} + \sin t\hat{j} + (-1)\hat{k}$ at $t = \dfrac{\pi}{4}$.

23. Apply divergence theorem to evaluate $\oiint_S \vec{F} \cdot \vec{n} dS$, where $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

24. Find the flux of $\vec{F} = (x - y)\hat{i} + x\hat{j} + 0\hat{k}$ across the circle $x^2 + y^2 = 1$ in the $xy-$plane.

25. Solve: $(x^2 + y^2 - a^2)xdx + (x^2 - y^2 - b^2)ydy = 0$.

26. Solve: $\sin^{-1} p = y - xp$.

27. The ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ is shifted 4 units to the right and 3 unit to up to generate the ellipse $\dfrac{(x-2)^2}{16} + \dfrac{(y-3)^2}{9} = 1$. Find the centre, focus, vertices of the new ellipse.

Part D
Each question carries 12 marks
Answer any two.

28. Find $\oiint_S \vec{F} \cdot \vec{n} dS$ where $\vec{F} = (2x + 3z)\hat{i} + (-xz - y)\hat{j} + (y^2 + 2z)\hat{k}$ and S is the surface of the sphere having centre at $(3, -1, 2)$ and radius 3.

29. Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region defined by $y = \sqrt{x}$ and $y = x^2$.

30. (a) Solve: $(\cos x \tan y + \cos(x + y))dx + (\sin x \sec^2 y + \cos(x + y))dy = 0$.

    (b) Solve: $(y - px)(p - 1) = p$.

31. Find the centre, foci, eccentricity and directorices of the conic $9x^2 - 18x - 16y^2 - 64y + 89 = 0$.