Reg. No .... Name **21P4040**

M. Sc DEGREE END SEMESTER EXAMINATION - APRIL 2021

SEMESTER 4 : MATHEMATICS

COURSE : **16P4MATT19EL : THEORY OF WAVELETS**

(For Regular - 2019 Admission and Supplementary - 2018/2017/2016 Admissions)

Time : Three Hours Max. Marks: 75

PART A
Answer All (1.5 marks each)

1. Define translation by k operator.
2. Prove that $(x * y) * z = x * (y * z)$ for all $x, y, z \in l^2(Z_N)$.
3. For any $z \in l^2(Z_N)$, prove that z is purely imaginary if and only if $\hat{z}(m) = -\hat{z}(N-m)$
4. If N is divisible by 2^p,define a p^{th}stage wavelet filter sequence. Hence define the system matrix $A_l(n)$.
5. With the usual notations express f_l and g_l interms of upsampling operators and convolutions.
6. Suppose N is divisible by 2^p. Suppose $u_l, v_l \in l^2(Z(\frac{N}{2}^{l-1})$ for $l = 1, 2, .., p$. Define $f_1 = v_1, g_1 = u_1$ and for $l = 2, 3, .., p$ define $f_l = g_{l-1} * U^{l-1}(v_l)$, prove that $f_l = u_1 * U(u_2) * U^2(u_3) * \ldots * U^{l-2}(u_{l-1}) * U^{l-1}(v_l)$.
7. Define the trigonometric system. Hence define a trigonometric polynomial. Is $sin(\theta - \theta_0)$ a trigonometric polynomial ? justify.
8. If z=$(z(n))_{n \in Z}$ is square summable and $\alpha \in C$, prove that αz ,is square summable.
9. For $z, w \in l^2(Z)$, define $z * w$.
10. Suppose $z, w \in l^2(Z)$ and $l \in N$. Then prove that $D^l(z) * w = D^l(z * U^l(w))$.

(1.5 x 10 = 15)

PART B
Answer any 4 (5 marks each)

11. Let $b \in l^2(Z_N)$ and $T_b : l^2(Z_N) \to l^2(Z_N)$ be defined by $T_b(z) = b * z$. Then prove that T_b is translation invariant linear transformation.
12. Suppose N is divisible by 2^l, x, y, $w \in l^2(Z^l_{N/2})$ and $z \in l^2(Z_N)$. Then prove that $D^l(z) * w = D^l(z * U^l(w)$ and $U^l(x * y) = U^l(x) * U^l(y)$.
13. Describe the analysis phase and synthesis phase in a filter bank diagram through an example.
14. Let H be a Hilbert space.
i) If $\{f_n\}_{n=1}^{\infty}$ is a sequence in H and $f \in H$, prove that $f_n \to f$ implies $< f_n, g > \to < f, g >$ for all $g \in H$.
ii) If $\{a_j\}_{j \in Z}$ is an orthonormal set in H and $z = (z(n))_{n \in Z} \in l^2(Z)$, prove that $< \sum\limits_{j \in Z} z(j) a_j, a_m > = z(m)$ for all $m \in Z$.
15. Prove that $L^2[(-\pi, \pi)]$ is a normed space.
16. (i) Define $f(\theta) = \frac{1}{4\sqrt{|\theta|}}$ when $\theta \neq 0$ and $f(0) = 0$. Prove that $f \in L^2([-\pi, \pi))$ but $f^2 \notin L^2([-\pi, \pi))$.
(ii) Prove that there exist $z, w \in l^2(Z)$ such that $z * w \notin l^2(Z)$.
(iii) Prove that $z * \delta = z$ for all $z \in l^2(Z)$.

(5 x 4 = 20)

PART C

Answer any 4 (10 marks each)

17.1. (a) State and prove the Fourier inversion formula
(b) Derive the parsevel's relation.
(c) From (b) deduce the Plancherel's formula.

OR

2. (a) Describe first stage shannon basis for $l^2(Z_N)$ if N is divisible by 4.
(b) Deduce the first stage real shannon basis for $l^2(Z_N)$.

18.1. Suppose N is divisible by $2^l, g_{l-1} \in l^2(Z_N)$ and the set $\{R_{2^{l-1}k} g_{l-1}\}_{k=0}^{\frac{N}{2^{l-1}}-1}$ is orthonormal and has $\frac{N}{2^{l-1}}$ elements. Suppose $u_l, v_l \in l^2(Z_{N/2}^{l-1})$ and the system matrix $A_l(n)$ is unitary for all $n = 0, 1, 2, \ldots, (N/2^l) - 1$. Define $f_l = g_{l-1} * U^{l-1}(v_l)$ and $g_l = g_{l-1}(*)U^{l-1}(u_l)$. With the usual notations prove that $V_{-l} \oplus W_{-l} = V_{-l+1}$

OR

2. Describe Haar wavelet system.

19.1. Suppose $f \in L^1([-\pi, \pi))$ and $< f, e^{in\theta} >= 0$ for all $n \in Z$. Then prove that $f(\theta)$=0 a.e
ii) Prove that the trigonometric system is complete in $L^2([-\pi, \pi))$.

OR

2. Suppose $T : L^2([-\pi, \pi)) \to L^2([-\pi, \pi))$ is a bounded translation invariant linear transformation . Then prove that $T(e^{im\theta}) = \lambda_m e^{im\theta}$ for some $\lambda_m \in C$ and it is true for all $m \in Z$.

20.1. Suppose $u, v \in l^1(Z)$. Then prove that $B = \{R_{2k}v\}_{k \in Z} \bigcup \{R_{2k}u\}_{k \in Z}$ is a complete orthonormal set in $l^2(Z)$ if and only if $A(\theta)$ is unitary for all $\theta \in [0, \pi)$.

OR

2. Let $p \in N$. For $l = 1, 2, .., p$, suppose $u_l, v_l \in l^1(Z)$ and the system matrix $A_l(\theta)$ is unitary for all $\theta \in [0, \pi)$. Define $f_1 = v_1, g_1 = u_1$, and for $l = 2, 3, 4, .., p$ define $f_l = g_{l-1} * U^{l-1}(v_l)$ and $g_l = g_{l-1} * U^{l-1}(u_l)$. Then prove that $f_1, f_2, \ldots, f_p, g_p$ generate a p^{th} stage wavelet system for $l^2(Z)$.

(10 x 4 = 40)