Reg. No .... Name **17P157**

M Sc DEGREE END SEMESTER EXAMINATION- NOVEMBER 2017

SEMESTER 1 : MATHEMATICS

COURSE : **16P1MATT05 ; COMPLEX ANALYSIS**

(Common for Regular - 2017 / Supplementary - 2016 Admissions)

Time : Three Hours Max. Marks: 75

Section A

Answer all the questions (1.5 marks each)

1. Prove that an anlytic function in a region Ω whose derivatives vanishes identically must reduce to a constant
2. Show that $w = iz + i$ maps half plane $x > 0$ onto the halfplane $v > 1$
3. Prove that the map $w = \bar{z}$ is not conformal
4. State Morera's theorem
5. Evaluate $\int_r xdz$, where r is the directed line seqment from 0 to $1 + i$
6. $\bar{r(t)} = t^2 e^{i\pi/4}, t \in (0, 1]$ is a non simple smooth contour. True or false. Justify
7. Find the types of singularities and their order of the function $\dfrac{1 + 2z^2}{z^3 + z^5}$
8. Define simply connected region with examples.
9. Find the poles and residues of the function $1/sinz$
10. Find the residue of $\dfrac{z + 1}{z^2(z - 3)}$ at $z = 0$

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. Find a bilinear transformation which maps the points $0, -i, -1$ onto the points $i, 1, 0$
12. Give a precise definition of a singlevalued branch of $loglogz$ and prove that it is analytic
13. Evaluate $\int_C \dfrac{e^{2z}dz}{(z + 1)^4}$ where C is$|z| = 3$
14. Show that the order of a zero of a polynomial equals the order of its first non-vanishing derivative.
15. Evaluate $\int_C \dfrac{sin\pi z^2 + cos\pi z^2}{(z - 1)^2(z - 2)}dz$, where C is the circle $|z| = 3$
16. State Rouche's theorem and apply it to determine the number of roots of the equation $z^8 - 4z^5 + z^2 - 1 = 0$

(5 x 4 = 20)

Section C

Answer either 1 OR 2 of each question (10 marks each)

17.1. Discuss the transformation $w = \frac{1}{z}$?. Also find the imagesof the infinite strips

$(i) 1/4 < y < 1/2$ $(ii) 0 < y < 1/2$

$(iii) 1/4 < x < 1/2$

OR

2. Find the Mobius transformation which maps the circle $|z| \leqq 1$ on $|w-1| \leqq 1$ and makes the points $z = 0, 1$ correspond to $w = 1/2, 0$ respectively

18.1. a) Evaluate $\int_C \frac{cos(e^z)}{z(z+2)} dz$ where $C = z : |z| = 1$, taken in the positive sense

b) Evaluate $\int_C \frac{sinhz}{z^2(z-2)} dz$, where $C = z : |z| = 1$, taken in the positive sense

OR

2. a. Evaluate $\int_{|z|=4} \frac{z^4}{(z-i)^3} dz$

b. State and prove Cauchy's Integral formula

19.1. State and prove Maximum principle

OR

2. Define a simply connected region with two examples. Also prove that a region Ω is simply connected iff $n(\Upsilon, a) = 0$ for all cycles Υ in Ω and all point a which do not belong to Ω

20.1. a. Evaluate $\int_0^\infty \frac{cosx}{x^2+a^2} dx$, a real

b. Evaluate $\int_- 0^\infty \frac{x^2}{(x^2+a^2)^3} dx$

OR

2. a. Evaluate $\int_{-\infty}^\infty \frac{x^2}{(x^2+a^2)^3} dx$

b. Evaluate $\int_0^\pi /2 \frac{dx}{a+sin^2 x} dx, |a| > 1$

(10 x 4 = 40)