Reg. No .... Name **20P3031**

M. Sc DEGREE END SEMESTER EXAMINATION - OCT/NOV 2020: JAN 2021

SEMESTER 3 : MATHEMATICS

COURSE : **16P3MATT13 : GRAPH THEORY**

(For Regular - 2019 Admission and Supplementary - 2016/2017/2018 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer all (1.5 marks each)

1. Define line graph of a loopless graph G.What is the line graph of $K_{1,n}$?
2. Give an example of a graph G and its subgraph H with the property $\lambda(G) = \lambda(H)$.
3. Prove or disprove: If closure of G is Hamiltonian, then G is Hamiltonian.
4. Give an example of a tree with two central vertices, one of which is also a centroidal vertex.
5. Determine $\tau(C_4)$
6. Draw the Petersen graph
7. Define proper vertex coloring and chromatic number of a graph G.
8. Define maximum matching and maximal matching in a graph. Give an example of a maximal matching which is not a maximum matching.
9. Explain the Jordan Curve Theorem.
10. Determine $\chi'(K_4)$

(1.5 x 10 = 15)

PART B

Answer any 4 (5 marks each)

11. Give an example of a graph on 10 vertices containing exactly 4 blocks.
12. Show that every connected graph contains a spanning tree.
13. (a) Show that a tree with at least two vertices contains at least two pendant vertices.
    (b) Show that if $\delta(G) \geq 2$, then G contains a cycle.
14. Briefly describe the Konigsberg Bridge problem and its significance.
15. Prove that for any graph G, $\alpha(G) + \beta(G) = n$
16. Show that $K_{3,3}$ is nonplanar.

(5 x 4 = 20)

PART C

Answer any 4 (10 marks each)

17.1. Define automorphism of a simple graph G. Show that the set $\Gamma(G)$ of all automorphisms of a simple graph G is a group with respect to the compositions of mappings as the group operation. Further show that for any simple graph G, $\Gamma(G) = \Gamma(G^c)$

OR

2. (a) Show that the automorphism group of K_3 is isomorphic to the symmetric S_3.
   (b) Define identity graph.Show that the graph G with vertex set $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and edge set $\{v_1v_4, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_7\}$ is an identity graph.

18.1. Find the number of spanning trees of the graph $C_3 \vee K_1$

OR

2. Show that every tree has a center consisting of either a single vertex or two adjacent vertices.

19.1. Show that a graph G is Eulerian if and only if it has an odd number of cycle decompositions.

OR

2. Show that for every positive integer k, there exists a triangle-free graph with chromatic number k.

20.1. Show that if G is a loopless bipartite graph, $\chi'(G) = \Delta(G)$.Show by means of an example that the converse need not be true.

OR

2. Show that every planar graph is 5-vertex colorable.

(10 x 4 = 40)