Reg. No .... Name **17P144**

M Sc DEGREE END SEMESTER EXAMINATION- NOVEMBER 2017

SEMESTER 1 : MATHEMATICS

COURSE : **16P1MATT04 ; ORDINARY DIFFERENTIAL EQUATIONS**

(Common for Regular - 2017 / Supplementary - 2016 Admissions)

Time : Three Hours Max. Marks: 75

Section A

Answer all the questions (1.5 marks each)

1. Is $(e^{-t}, -e^{-t})$ a solution of $\frac{dx}{dt} = 3x + 4y$. Justify your answer.

$$\frac{dy}{dt} = 2x + y$$

2. Find any one characteristic vector of the matrix $\begin{bmatrix} 3 & 1 \\ 12 & 2 \end{bmatrix}$.
3. Is there exist any homogeneous linear system of two unknown functions on an interval $-1 \le t \le 1$ such that its wronskian of two solutions is W(t) = t on $-1 \le t \le 1$. Justify your answer.
4. Define Cauchy product of two series and state a necessary condition for the cauchy product of two convergent series to be convergent.
5. Give an example of two convergent series $\sum_{j=0}^{\infty} a_j$ and $\sum_{j=0}^{\infty} b_j$ such that $\sum_{j=0}^{\infty} a_j b_j$ is not convergent.
6. The sequence of functions $\{cos(nx)\}_{n=1}^{\infty}$ is orthonormalized with respect to the weight function $r(x) = 1$ on the interval $0 \le x \le \pi$. State true or false and justify your answer.
7. Cosider the sequence of fucntions $\{\varphi_n(x)\}_{n=1}^{\infty}$ where $\varphi_n(x) = k_n sin(nx)$. Find $k_n(n = 1, 2, 3, \dots\dots)$ such that the given sequence is orthonormal with respect to the weight function $r(x) = 1$ on the interval $0 \le x \le \pi$.
8. Find $L[4sin(x)cos(x) + 2e^{-x}]$.
9. Find a function f whose Laplace transform is $\frac{2}{p+3}$.
10. Find $L[x^2 sin(ax)]$.

(1.5 x 10 = 15)

Section B

Answer any 4 (5 marks each)

11. Find all characteristic values and vectors of the matrix $\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$.
12. Find the general solution of the system $\frac{dx}{dt} = 3x + y, \frac{dy}{dt} = 4x + 3y$.
13. Use Picard's method to solve the initial value problem $y' = 2y, y(0) = 1$.

14. Explain Strum - Liouville problem and properties of its characteristic values and characteristic functions.

15. Use the Laplace transform to solve the integral equation $y(x) = 1 - \int_0^x (x-t)y(t)dt$.

16. Show that the differential equation $y'' + a^2y = f(x), y(0) = y'(0) = 0$ has solution $y(x) = \frac{1}{a}\int_0^x f(t)sina(x-t)dt$

(5 x 4 = 20)

Section C

Answer either 1 OR 2 of each question (10 marks each)

17.1. Solve $t\frac{dx}{dt} = x + y$

$$t\frac{dy}{dt} = -3x + 5y$$

OR

2. Find all characteristic values and vectors of the matrix $\begin{bmatrix} -2 & 5 & 5 \\ -1 & 4 & 5 \\ 3 & -3 & 2 \end{bmatrix}$.

18.1. Let p be an arbitrary real constant. Use a differential equation to derive the power series expansion for the function $y = (1+x)^p$.

OR

2. Verify that 0 is an ordinary point and then find the power series solution of the differential equation $(1+x^2)y'' + xy' + y = 0$.

19.1. Consider the Strum-Liouville problem $\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right] + [q(x) + \lambda r(x)]\,y = 0$ with boundary conditions $A_1y(a) + A_2y'(a) = 0$ and $B_1y(b) + B_2y'(b) = 0$ where A_1, A_2, B_1, B_2 are real constants such that A_1 and A_2 are not both zero and B_1 and B_2 are not both zero. Show that the characteristic funcions corresponding to distinct characteristic values are orthogonal with respect to the weight function $r(x)$ on the interval $a \le x \le b$.

OR

2. Find characteristic functions of the Strum - Liouville problem $\frac{d^2y}{dx^2} + \lambda y = 0, y(0) = 0, y(\pi/2) = 0$ and show that the characteristic funcions corresponding to distinct characteristic values are orthogonal with respect to the weight function $r(x) = 1$ on the interval $0 \le x \le \pi$.

20.1. Use Laplace transform to solve the differential equation $xy'' + (2x+3)y' + (x+3)y = 3e^{-x}$ with initial condition $y(0) = 0$.

OR

2. Use the principle of superposition to solve the equation $y'' + 5y' - 6y = 5e^{3t}$ with initial conditions y(0)=0, y'(0)=0.

(10 x 4 = 40)