Reg. No .... Name **21P4028**

M. Sc DEGREE END SEMESTER EXAMINATION - APRIL 2021

SEMESTER 4 : MATHEMATICS

COURSE : **16P4MATT18EL : COMBINATORICS**

(For Regular - 2019 Admission and Supplementary - 2018/2017/2016 Admissions)

Time : Three Hours Max. Marks: 75

PART A

Answer All (1.5 marks each)

1. Explain the mutiplication principle?
2. Find the number of ways of arranging the 26 letters in the English alphabet in a row such that there are exactly 5 letters between x and y.
3. How many 5-letter words can be formed using A, B, C, D, E, F, G, H, I, J, ; if the letters in each word must be distinct?
4. Explain The Pigeonhole Principle?
5. Define Ramsey's theorem.
6. State the Generalized form of Ramsey number.
7. Explain principle of Inclusion and Exclusion
8. Find the primes between 2 and 48 inclusive by the Sieve of Eratosthenes?
9. Find the generating function for the sequence $\left(\binom{n}{0}, \binom{n}{1}, \ldots\ldots, \binom{n}{n}, 0, 0.....\right)$.
10. Let (a_r) denote the number of r-permutations of the multi-set $M = \{\infty.\, b_1, \infty.\, b_2, \ldots, \infty.\, b_k\}$Then find the exponential generating function for (a_r)

(1.5 x 10 = 15)

PART B

Answer any 4 (5 marks each)

11. a) Find the number of ternary sequences of length 10 having two O's, three 1's and five 2's.?
b) Find the number of ways to pave a 1 x 7 rectangle by 1 x 1, 1 x 2 and 1 x 3 blocks, assuming that blocks of the same size 'are indistinguishable.
12. a) Prove that P(r,r_1,r_2,......r_n) = $\frac{r!}{r_1!r_2!....r_n!}$.
b) Find the number of permutations of the 5 letters: a,a,a,b,c?
13. Seventeen people correspond by mail with one another - each one with all the rest. In their letters only three different topics are discussed. Each pair of correspondents deals with only one of these topics. Prove that there are at least three people who write to one another about the same topic.
14. Let $n \epsilon N$,and let $n = p_1^{m_1} p_2^{m_2} \ldots p_k^{m_k}$, be its prime fractorization then show that $\varphi(n) = n \prod_{i=1}^{k} \left(1 - \frac{1}{p_i}\right)$.
15. Find the number of integer solutions to the equation $x_1 + x_2 + x_3 = 28$,where $3 \leq x_1 \leq 9, 0 \leq x_2 \leq 8, 7 \leq x_3 \leq 17$; using properties?
16. Let a_r be the number of ways of distributing r identical objects into n distinct boxes such that no box is empty. Find the generating function for (a_r)?

(5 x 4 = 20)

PART C
Answer any 4 (10 marks each)

17.1. a) Find the number of ways to choose a pair {a, b} of distinct numbers from the set {1,2, ... ,50} such that
(i) | a - b | = 5;
(ii) | a - b | $\leq$ 5
b) There are 12 students in a party. Five of them are girls. In how many ways can these 12 students be arranged in a row if
(i) there are no restrictions?
(ii) the 5 girls must be together (forming a block)?
(iii) no 2 girls are adjacent?
(iv) between two particular boys A and B, there are no boys but exactly 3 girls?

OR

2. a) Explain stirling no.of 2nd kind and derive the recurrence relation.?.
b) Find the number of positive integer solutions to the equation; $3x_1 + 5x_2 + x_3 + x_4 = 10$

18.1. a) Ten players took part in a round robin chess tournament (i.e., each player must play exactly one game against every other player). According to the rules, a player scores 1 point if he wins a game; -1 point if he loses; and 0 point if the game ends in a draw. When the tournament was over, it was found that more than 70% of the games ended in a draw. Show that there were two players who had the same total score.?
b) Let A = {a_1, a_2, ... , a_5} be a set of 5 positive integers. Show that for any permutation $a_{i1}, a_{i2}, a_{i3}, a_{i4}, a_{i5}$ of A, the product.
$(a_{i1} - a_1)(a_{i2} - a_2)\ldots\ldots\ldots(a_{i5} - a_5)$ is always even.

OR

2. Let ABC be an equilateral triangle and ϵ the set of all points contained in the 3 segments AB, BC, CA (including A, B and C). Show that, for every partition of ϵ into 2 disjoint subsets, at least one of the 2 subsets contains the vertices of a right-angled triangle.

19.1. Derive the formula for F(n,m); where n, m ϵN, denote the number of surjective mappings from N_n to N_m? .

OR

2. Explain Distribution problems using generating functions

20.1. a) Solve the recurrence relation $a_n - 7a_{n-1} + 15a_{n-2} - 9a_{n-3} = 0$; Given that $a_0 = 1$, $a_1 = 2$ and $a_2 = 3$.
b)Solve the recurrence relation $a_n = 2(a_{n-1} - a_{n-2})$; Given that $a_0 = 1$ and $a_1 = 0$.

OR

2. The n sectors, $n \geq 1$, of the circle are to be coloured by k distinct colours, where $k \geq 3$, in such a way that each sector is coloured by one colour and any two adjacent sectors must be coloured by different colours. Let a_n denote the number of ways this can be done.
(i) Evaluate a_1, a_2 and a_3.
(ii) Find a recurrence relation for (a_n), $n \geq 4$, and solve the recurrence relation.

(10 x 4 = 40)